Introduction to calculus - Topic test 2

Name: _____ Date: _____

10 questions | 52 marks | 65 minutes

Move between numerical data, derivative information and polynomial graphs, then interpret models.

A scientific calculator may be used.

1. A position table gives $s(0)=3$, $s(1)=5$, $s(2)=11$ and $s(3)=21$ metres. [5 marks]

Position measurements

t (s)	0	1	2	3
s (m)	3	5	11	21

(a) Find the average velocities on $[0,1]$, $[1,2]$ and $[2,3]$. [3 marks]

(b) Use $[1,2]$ to estimate velocity near 1.5s. Is it necessarily exact? [2 marks]

2. Use $f(x)=x^3$ at $x=1$. [4 marks]

(a) Simplify its difference quotient. [3 marks]

(b) Find the instantaneous gradient. [1 mark]

3. A quadratic is $f(x)=ax^2+bx+c$, with $f'(0)=-4$, $f'(2)=8$ and $f(0)=5$. [4 marks]

(a) Find a and b. [3 marks]

(b) Write the full function. [1 mark]

4. For $y = x^3$, [6 marks]

(a) find the normal at $x = 1$. [3 marks]

(b) find all points where the tangent gradient is 3. [3 marks]

5. Compare $f(x) = x^3$ and $g(x) = -x^4$ at $x = 0$. [5 marks]

(a) Show that each has a stationary point there. [2 marks]

(b) Classify each using signs. [3 marks]

6. Analyse $f(x) = x^3 - 6x^2 + 9x$. [7 marks]

(a) Find all intercepts and stationary coordinates. [4 marks]

(b) Give monotonic intervals and stationary types. [3 marks]

7. $s(t) = t^3 - 3t^2$ metres on $0 \leq t \leq 4$ seconds. [6 marks]

(a) Find rest times and the interior change of direction. [3 marks]

(b) Find displacement and total distance. [3 marks]

8. A garden against a wall has 48m of fencing for the other three sides. [5 marks]

(a) Form its area in terms of the perpendicular side x. [2 marks]

(b) Find the greatest area. [3 marks]

9. Cost is $C(x) = 0.25x^2 - 5x + 90$ on $0 \le x \le 30$. [5 marks]

(a) Find and classify its stationary point. [3 marks]

(b) Check that it is the global minimum on this domain. [2 marks]

10. A quadratic has derivative $f'(x) = 2x - 4$ and $f(1) = 7$. [5 marks]

(a) Find f by comparing quadratic coefficients. [3 marks]

(b) Find its minimum value. [2 marks]