Introduction to calculus - Topic test 1

Name: _____ Date: _____

10 questions | 48 marks | 65 minutes

A balanced first paper: rates, derivative rules, tangent lines, graph behaviour, optimisation and motion.

A scientific calculator may be used.

1. A position model is $s(t) = t^2 + 1$ metres. [4 marks]

(a) Find the average velocity from 1s to 3s. [2 marks]

(b) Compare this with the average from 0s to 2s and interpret the difference. [2 marks]

2. For $f(x) = x^2$, investigate the gradient at $x = 2$. [4 marks]

(a) Simplify $\frac{f(2+h)-f(2)}{h}$ for $h \neq 0$. [2 marks]

(b) State the limiting gradient and explain why $h = 0$ is not substituted first. [2 marks]

3. Let $f(x) = 2x^3 - 3x^2 + 4$. [4 marks]

(a) Find its derivative function. [2 marks]

(b) Find $f'(2)$ and distinguish it from $f(2)$. [2 marks]

4. Consider $y = x^2 - 2x + 3$ at $x = 2$. [5 marks]

(a) Find the tangent equation. [3 marks]

(b) Find the normal equation. [2 marks]

5. For $f(x) = x^3 - 3x$, [6 marks]

(a) find intervals of increase and decrease. [3 marks]

(b) find and classify both stationary points. [3 marks]

6. You are given $g'(x) = -(x+2)(x-1)$. [5 marks]

(a) Construct a sign chart. [2 marks]

(b) State the stationary types and explain what is needed to find their heights. [3 marks]

7. A rectangle has perimeter 32m. Let its width be x metres. [5 marks]

(a) Form an area function and state its physical domain. [2 marks]

(b) Find and justify its maximum area. [3 marks]

8. Maximise $f(x) = -x^2 + 6x$ over $0 \leq x \leq 2$. [4 marks]

(a) Find its stationary input and say whether it is allowed. [2 marks]

(b) Determine the maximum on the given interval. [2 marks]

9. $s(t) = t^2 - 4t$ metres on $0 \leq t \leq 5$ seconds. [6 marks]

(a) Find the rest time and the direction on either side. [3 marks]

(b) Find displacement and total distance. [3 marks]

10. Revenue is $R(x) = x(20 - x)$ dollars and cost is $C(x) = 4x + 12$, for $0 \leq x \leq 15$. [5 marks]

(a) Form the profit function. [2 marks]

(b) Find the greatest modelled profit and the corresponding x. [3 marks]