Simple optimisation

6 questions | 26 marks

Equivalent correct methods earn the corresponding marks.

1. A rectangle has perimeter 24m. Find its maximum area. [4 marks]

$A = x(12 - x)$; $A' = 12 - 2x = 0$ at 6.

The downward parabola gives the maximum.

Answer: 36m² with sides 6m and 6m.

2. A rectangle beside a wall has 36m of fencing for the other three sides. Find its maximum area. [4 marks]

$A = x(36 - 2x)$; $A' = 36 - 4x = 0$ gives 9.

The other side is 18m and area 162m².

Answer: 162m²; perpendicular sides 9m and remaining side 18m.

3. Maximise $P(x) = -2x^2 + 24x - 10$ on $[0, 10]$. [4 marks]

$P' = -4x + 24 = 0$ gives 6.

Derivative signs + then − establish the maximum; $P(6) = 62$.

Answer: Maximum 62 at $x = 6$.

4. Minimise $C(x) = x^2 - 10x + 40$ on $[0, 8]$. [4 marks]

$C' = 2x - 10 = 0$ gives 5.

Compare $C(0) = 40$, $C(5) = 15$ and $C(8) = 24$.

Answer: Minimum 15 at $x = 5$.

5. Maximise $f(x) = 100 - (x - 8)^2$ on $[0, 6]$. [4 marks]

$f' = 16 - 2x$ is positive throughout the domain.

The stationary input 8 is outside the interval; $f(6) = 96$ is greatest.

Answer: 96 at $x = 6$.

6. An open box is made by removing squares of side x cm from each corner of a 12cm square, then folding. Find the greatest volume. [6 marks]

$V = x(12 - 2x)^2 = 144x - 48x^2 + 4x^3$, with $0 < x < 6$.

$V' = 12(x - 2)(x - 6)$, so the admissible stationary input is 2.

V rises then falls and tends to 0 at both boundaries; $V(2) = 128$.

Answer: 128cm³ at $x = 2$ cm.