First-derivative graph analysis

6 questions | 24 marks

Equivalent correct methods earn the corresponding marks.

1. List intercepts, stationary point and monotonic intervals of $f(x) = x^2 - 4x + 3$. [5 marks]

$f = (x-1)(x-3)$; $f' = 2x - 4$.

Evaluate f at 0 and 2 and use derivative signs.

Answer: Roots 1,3; y-intercept 3; minimum $(2, -1)$; decreasing $x < 2$, increasing $x > 2$.

2. Analyse $f(x) = x^3 - 3x^2$. [5 marks]

$f = x^2(x-3)$ and $f' = 3x(x-2)$.

The derivative signs are +, −, +.

Answer: Intercepts $(0,0), (3,0)$; maximum $(0,0)$; minimum $(2, -4)$.

3. A differentiable function has derivative zero at $x = -2$ and $x = 1$. Its derivative is positive for $x < -2$, negative for $-2 < x < 1$, and positive for $x > 1$. State the stationary types. [3 marks]

The signs change + to − at -2 and − to + at 1.

No heights can be found from this information alone.

Answer: Maximum at $x = -2$; minimum at $x = 1$.

4. Find roots and stationary points of $g(x) = x^3 - 3x$. [5 marks]

$g = x(x^2 - 3)$ and $g' = 3(x - 1)(x + 1)$.

Use signs +, −, + and evaluate g at ± 1.

Answer: Roots $0, \pm\sqrt{3}$; maximum $(-1, 2)$, minimum $(1, -2)$.

5. A straight-line function has $f'(x) = 2$ and $f(0) = -3$. Find f. [3 marks]

A line of gradient 2 has form $2x + c$.

The intercept condition makes $c = -3$.

Answer: $f(x) = 2x - 3$

6. Why can the derivative alone not determine vertical position? [3 marks]

For example x^2 and $x^2 + 5$ both have derivative $2x$ but different heights.

Answer: Adding a constant leaves the derivative unchanged.