Instantaneous rates and limiting secants

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. Use a difference quotient for $f(x) = x^2$ at 1. [3 marks]

$\frac{(1+h)^2 - 1}{h} = 2 + h$ for $h \neq 0$.

The limit is 2.

Answer: 2

2. Use a difference quotient for $f(x) = 3x^2$ at 2. [3 marks]

$\frac{3(2+h)^2 - 12}{h} = 12 + 3h$.

Its limit is 12.

Answer: 12

3. Find the gradient of $f(x) = x^3$ at 1 using its difference quotient. [3 marks]

$\frac{(1+h)^3 - 1}{h} = 3 + 3h + h^2$.

The limit is 3.

Answer: 3

4. For $f(x)=x^2$ at 2, calculate secant gradients for $h=1, 0.1, 0.01$ and state the limiting value. [3 marks]

The quotient is $4+h$.

Substitute the three h-values and then let h approach 0.

Answer: 5, 4.1, 4.01; limit 4

5. For $f(x)=|x|$ at 0, compare the left and right difference quotients. Does one derivative exist? [3 marks]

$\frac{|h|}{h}$ equals 1 for positive h and -1 for negative h.

The two limiting slopes disagree.

Answer: Left slope -1, right slope 1; no derivative.

6. $s(t)=5t^2$ metres. Use a limiting quotient to find velocity at 3 seconds. [3 marks]

$\frac{5(3+h)^2-45}{h}=30+5h$.

The limit is 30 metres per second.

Answer: 30 m/s