Average rates and secant gradients

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. Find the average rate of $f(x) = x^2$ from 0 to 4. [3 marks]

$f(0) = 0,\ f(4) = 16.$

$\frac{16-0}{4-0} = 4.$

Answer: 4

2. Find the secant gradient of $g(x) = x^3$ between 1 and 3. [3 marks]

$g(1) = 1,\ g(3) = 27.$

$\frac{27-1}{3-1} = 13.$

Answer: 13

3. A population rises from 320 to 395 over 5 years. Find its average yearly increase. [3 marks]

The change is $395 - 320 = 75$ people.

Divide by 5 years: $\frac{75}{5} = 15.$

Answer: 15 people per year

4. $C(t) = 100 - 4t$. Find its average rate from 2 to 7. [3 marks]

$C(2) = 92,\ C(7) = 72.$

$\frac{72-92}{7-2} = -4.$

Answer: -4 units per time unit

5. A secant passes through $(1, 5)$ and $(4, 17)$. Find its gradient. [3 marks]

$\Delta y = 12,\ \Delta x = 3.$

$m = \frac{12}{3} = 4.$

Answer: 4

6. $h(t) = t^2 - 4t$ on $0 \le t \le 4$. Its average rate is 0. Must its instantaneous rate always be 0? Explain. [3 marks]

$h(0) = h(4) = 0$, but $h(2) = -4$.

Equal endpoints do not imply a constant graph.

Answer: No. It falls and rises back to the starting value.