# Exponential and logarithmic functions - Applications test

Name: _____ Date: _____

10 questions | 53 marks | 75 minutes

Apply exact algebra and logarithmic models to medicine, finance, calibrated scales and the reliability of forecasts.

A scientific calculator may be used.

**1.** Calculate exact values and handle signs carefully. [4 marks]

**(a)** Evaluate $125^{\frac{2}{3}}$ and $16^{-\frac{1}{2}}$. [2 marks]

**(b)** Simplify $\sqrt{x^2}$ for $x = -7$ and explain why it is not $x$. [2 marks]

**2.** Work exactly before rounding. [4 marks]

**(a)** Rationalise $\frac{6}{\sqrt{7}-1}$. [2 marks]

**(b)** Find its value to three decimal places. [2 marks]

**3.** Two laboratory quantities, in milligrams, follow $A(t)=120(1.25)^t$ and $B(t)=300(0.9)^t$, where $t$ is hours. Assume both rules are valid for $0 \le t \le 5$. [5 marks]

**(a)** Form an equation for the time when the amounts are equal. [2 marks]

**(b)** Solve the equation and identify which amount is larger at $t=0$. [3 marks]

**4.** A medicine model is $M(t) = 48(0.5)^{\frac{t}{4}}$ mg, where $t$ is hours after one dose. Assume no further doses. [5 marks]

**(a)** State its half-life and calculate $M(10)$. [2 marks]

**(b)** Find all times $t \geq 0$ for which the model predicts less than 5 mg. [3 marks]

**5.** A transformed exponential has horizontal asymptote $y = 2$ and passes through $(0, 5)$ and $(1, 11)$. Its rule is $y = k + Ab^x$. [5 marks]

**(a)** Find $k$, $A$ and $b$. [3 marks]

**(b)** Predict its value at $x = 2$ and state its range. [2 marks]

**6.** A scale is defined by $p = -\log C$, where $C$ is a positive concentration in a fixed unit. [6 marks]

**(a)** Find $C$ if $p = 4.5$. [3 marks]

**(b)** If $p$ increases from 4.5 to 6.5, by what factor does $C$ change? [3 marks]

**7.** A sound-level model is $L = 10\log(\frac{I}{I_0})$, where $I$ and the fixed reference $I_0$ are positive intensities. [5 marks]

**(a)** Find the increase in $L$ when $I$ is multiplied by 100. [2 marks]

**(b)** A level rises by 3 units. Find the intensity multiplier to three decimals. [3 marks]

**8.** An initial 6000 dollars earns a nominal 4.8% per annum compounded monthly, with no fees or extra payments. [6 marks]

**(a)** Write the balance after $m$ complete months and calculate it after 18 months. [3 marks]

**(b)** Find the first complete month in which the balance exceeds 7000 dollars. [3 marks]

**9.** Solve $2\log(x-1)=\log(3x+1)$. [5 marks]

**(a)** State the original domain and obtain a polynomial equation. [3 marks]

**(b)** Find the valid root and verify it. [2 marks]

**10.** A researcher fits the first three equally spaced observations below with a pure exponential model. [8 marks]

Observed quantity

| Time $t$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Observed value | 50 | 75 | 112.5 | 160 |

**(a)** Find that model and its prediction at $t = 3$. [3 marks]

**(b)** The actual $t = 3$ observation is 160. Find the absolute error and the percentage error relative to the actual value. [3 marks]

**(c)** Explain why this fourth observation matters before forecasting much further. [2 marks]