Exponential and logarithmic functions - Practice test 4

Name: _____ Date: _____

10 questions | 48 marks | 75 minutes

Test the reasoning behind algebraic rules, domains, graph features and model selection.

A scientific calculator may be used.

1. Consider the definition of a real logarithm. [4 marks]

(a) State the conditions on base b and argument x for $\log_b x$. [2 marks]

(b) Use the definition to evaluate $\log_{\frac{1}{3}} 9$. [2 marks]

2. An asset gains 20% in one year and then loses 20% the following year. [5 marks]

(a) Find the combined multiplier and overall percentage change. [2 marks]

(b) After a 20% loss, what percentage increase restores the original value? [3 marks]

3. Disprove each statement with an explicit numerical counterexample. [4 marks]

(a) $\log(a+b) = \log a + \log b$ for all positive a, b. [2 marks]

(b) $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ for all non-negative a, b. [2 marks]

4. Consider $f(x) = 12 - 3(4^{x-2})$. [5 marks]

(a) Find its y-intercept, horizontal asymptote and range. [3 marks]

(b) Find the exact x-intercept. [2 marks]

5. Solve $2^{2x} - 5(2^x) + 4 = 0$. [5 marks]

(a) Use a substitution to obtain a quadratic. [2 marks]

(b) Find all real x. [3 marks]

6. A positive exponential model passes through $(1, 12)$ and $(3, 108)$. [5 marks]

(a) Find its rule. [3 marks]

(b) Explain why the negative square root for b is not used. [2 marks]

7. Compare equations that appear similar. [5 marks]

(a) Solve $\log(x^2) = 2$. [2 marks]

(b) Solve $2\log x = 2$ and explain the difference. [3 marks]

8. Solve $\log_{0.5} x > 3$. [5 marks]

(a) State the domain and solve the inequality. [3 marks]

(b) Check one value inside and one outside your solution interval. [2 marks]

9. A store fits exponential demand using observations $D(1) = 240$ and $D(2) = 288$. [4 marks]

(a) Find $D(0)$ under the fitted rule. [2 marks]

(b) A student uses 240 as the initial value in $D(t) = 240(1.2)^t$. Explain the error. [2 marks]

10. A school wants a model for the number of students in a room over a lesson. Arrivals stop when the room is full. [6 marks]

(a) Explain why observing rapid initial growth is not enough to justify unbounded exponential growth. [3 marks]

(b) Describe two checks you would make before using an exponential forecast. [3 marks]