Exponential and logarithmic functions - Practice test 4

10 questions | 48 marks | 75 minutes

Test the reasoning behind algebraic rules, domains, graph features and model selection.

Equivalent correct methods earn the corresponding marks.

1. Consider the definition of a real logarithm. [4 marks]

(a) State the conditions on base b and argument x for $\log_b x$. [2 marks]

A valid real exponential base must be positive and non-unit; its outputs are positive.

Answer: $b > 0$, $b \neq 1$, and $x > 0$.

(b) Use the definition to evaluate $\log_{\frac{1}{3}} 9$. [2 marks]

$(\frac{1}{3})^{-2} = 9$.

Answer: -2

2. An asset gains 20% in one year and then loses 20% the following year. [5 marks]

(a) Find the combined multiplier and overall percentage change. [2 marks]

$1.2(0.8) = 0.96$.

Answer: 0.96; 4% decrease.

(b) After a 20% loss, what percentage increase restores the original value? [3 marks]

The required restoring factor is $\frac{1}{0.8} = 1.25$.

Answer: 25%.

3. Disprove each statement with an explicit numerical counterexample. [4 marks]

(a) $\log(a+b)=\log a+\log b$ for all positive a, b. [2 marks]

$\log(1+1)=\log 2\neq 0=\log 1+\log 1$.

Answer: For $a=b=1$, the left side is $\log 2$, while the right side is 0.

(b) $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$ for all non-negative a, b. [2 marks]

$\sqrt{25}=5$ but $3+4=7$.

Answer: For $a=9,\ b=16$, the sides are 5 and 7.

4. Consider $f(x)=12-3(4^{x-2})$. [5 marks]

(a) Find its y-intercept, horizontal asymptote and range. [3 marks]

$f(0)=12-\frac{3}{16}=11.8125$; the subtracted term is always positive.

Answer: $(0, 11.8125)$; $y=12$; outputs less than 12.

(b) Find the exact x-intercept. [2 marks]

$4^{x-2}=4$, so $x-2=1$ and $x=3$.

Answer: $(3, 0)$

5. Solve $2^{2x}-5(2^x)+4=0$. [5 marks]

(a) Use a substitution to obtain a quadratic. [2 marks]

$2^{2x}=(2^x)^2$.

Answer: Let $u=2^x>0$; then $u^2-5u+4=0$.

(b) Find all real x. [3 marks]

$(u-1)(u-4)=0$ gives $u=1$ or 4.

Both are positive; solve $2^x=1$ or $2^x=4$.

Answer: $x=0$ or $x=2$.

6. A positive exponential model passes through $(1, 12)$ and $(3, 108)$. [5 marks]

(a) Find its rule. [3 marks]

$b^2 = \frac{108}{12} = 9$, so $b = 3$; $A = \frac{12}{3} = 4$.

Answer: $y = 4(3^t)$

(b) Explain why the negative square root for b is not used. [2 marks]

A negative base would not define a real-valued exponential function for every real t.

Answer: The model uses a positive real exponential base.

7. Compare equations that appear similar. [5 marks]

(a) Solve $\log(x^2) = 2$. [2 marks]

Original domain is $x \neq 0$; $x^2 = 100$ admits both signs.

Answer: $x = 10$ or $x = -10$.

(b) Solve $2\log x = 2$ and explain the difference. [3 marks]

$\log x = 1$ implies $x = 10$.

The identity with $2\log x$ assumes positive x.

Answer: $x = 10$ only; its original domain is $x > 0$.

8. Solve $\log_{0.5} x > 3$. [5 marks]

(a) State the domain and solve the inequality. [3 marks]

The base lies between zero and one, so its logarithm is decreasing.

$x < (0.5)^3 = \frac{1}{8}$, with $x > 0$.

Answer: $0 < x < \frac{1}{8}$

(b) Check one value inside and one outside your solution interval. [2 marks]

The logs are 4 and 2, respectively.

Answer: For example $x = \frac{1}{16}$ works; $x = \frac{1}{4}$ does not.

9. A store fits exponential demand using observations $D(1) = 240$ and $D(2) = 288$. [4 marks]

(a) Find $D(0)$ under the fitted rule. [2 marks]

The multiplier is $\frac{288}{240} = 1.2$; divide 240 by 1.2 to return to time zero.

Answer: 200

(b) A student uses 240 as the initial value in $D(t) = 240(1.2)^t$. Explain the error. [2 marks]

Its prediction at $t = 1$ is 288 rather than the stated 240.

Answer: It treats the time-one observation as time zero.

10. A school wants a model for the number of students in a room over a lesson. Arrivals stop when the room is full. [6 marks]

(a) Explain why observing rapid initial growth is not enough to justify unbounded exponential growth. [3 marks]

A constant multiplier cannot keep applying after arrivals stop at capacity.

Answer: A finite room capacity changes the future process.

(b) Describe two checks you would make before using an exponential forecast. [3 marks]

Use further observations, not only two points.

Confirm that the assumed multiplier remains plausible before the capacity limit.

Answer: Check ratios over equal time intervals and check the time range/physical assumptions.