Exponential and logarithmic functions - Practice test 3

Name: _____ Date: _____

10 questions | 50 marks | 75 minutes

Develop and compare models with different time periods, background levels and practical thresholds.

A scientific calculator may be used.

1. A function is $f(x) = 5(4^x)$. [5 marks]

(a) Find $f(0)$ and the range. [2 marks]

(b) Make x the subject of $y = f(x)$, and state the domain of the resulting inverse rule. [3 marks]

2. For this question x is positive. [4 marks]

(a) Simplify $(81x^{12})^{\frac{1}{4}}$. [2 marks]

(b) Simplify $\frac{\sqrt{45x^2}}{\sqrt{5}}$. [2 marks]

3. A cooling model is $T(t) = 18 + 62(0.85)^t$, where T is in degrees Celsius and t is in minutes. [6 marks]

(a) Find the initial temperature and explain the meaning of 18. [2 marks]

(b) For which times does the model predict a temperature below 25 degrees Celsius? [4 marks]

4. A culture starts with 300 cells and doubles every 30 minutes. Assume the model remains useful for the period considered. [5 marks]

(a) Write a model with t measured in hours. [2 marks]

(b) Predict its size after 75 minutes. [3 marks]

5. Account A pays a nominal 5.8% per annum compounded monthly. Account B pays 5.9% compounded annually. Both have no fees. [5 marks]

(a) Find the effective annual rate for A. [3 marks]

(b) For the same initial balance, which account has the larger balance after one year? Explain. [2 marks]

6. Solve, retaining only valid real solutions. [5 marks]

(a) $\log_2(x-1)=3$. [2 marks]

(b) $\log(x-1)+\log(x+1)=\log 24$. [3 marks]

7. A radioactive sample has 80 g initially and 10 g after 15 days. Model its decay using a fixed half-life. [5 marks]

(a) Find its half-life. [2 marks]

(b) Find when 1 g remains, to two decimals. [3 marks]

8. An observer records $y = 0$ at $t = 0$ and $y = 5$ at $t = 1$. [5 marks]

(a) Can $y = Ab^t$ with $b > 0$ fit both points? Explain. [3 marks]

(b) Give one alternative type of model that can fit both points, with an example. [2 marks]

9. A quantity is modelled by $Q(t) = 20(2^t)$ and is checked only after completed whole hours. [4 marks]

(a) In which hour does it first reach at least 640? [2 marks]

(b) In which hour does it first exceed 640? [2 marks]

10. Over a short interval, the radius of a growing sphere follows $r(t) = 10(1.03)^t$ centimetres. Use $V = \frac{4\pi r^3}{3}$. [6 marks]

(a) Write an exponential formula for its volume. [3 marks]

(b) Find its percentage volume increase per time unit. [3 marks]