Exponential and logarithmic functions - Practice test 2

10 questions | 46 marks | 70 minutes

Interpret tables, transformed curves and inverse relationships before analysing two competing models.

Equivalent correct methods earn the corresponding marks.

1. Use exact forms before decimal approximations. [4 marks]

(a) Evaluate $64^{\frac{2}{3}}$ and $81^{-\frac{3}{4}}$. [2 marks]

$\sqrt[3]{64}=4$, then square; $\sqrt[4]{81}=3$, then cube and invert.

Answer: 16 and $\frac{1}{27}$.

(b) Simplify $\sqrt{48}+2\sqrt{27}$. [2 marks]

$4\sqrt{3}+6\sqrt{3}=10\sqrt{3}$.

Answer: $10\sqrt{3}$

2. Decide whether each claim is valid and justify it. [4 marks]

(a) $\sqrt{x^2}=x$ for every real x. [2 marks]

For $x=-3$, the left side is 3 and the right side is -3.

Answer: False; the left side is the absolute value of x.

(b) $\log(2x)=\log 2+\log x$ for $x>0$. [2 marks]

It is the product law with positive arguments and matching bases.

Answer: True.

3. Use the equally spaced observations in the table. [4 marks]

Daily observations

t (days)	0	1	2
Amount	160	120	90

(a) Find a suitable exponential rule. State the daily percentage change. [2 marks]

$\frac{120}{160} = \frac{90}{120} = 0.75$; retain 75% each day.

Answer: $A(t) = 160(0.75)^t$; 25% daily decrease.

(b) Find the predicted amount at $t = 4$. [2 marks]

$160(0.75)^4 = 50.625$.

Answer: 50.625

4. Consider $g(x) = 7 - 3(2^{x+1})$. [6 marks]

(a) Find its y-intercept, asymptote and range. [3 marks]

$g(0) = 7 - 3(2) = 1$. The subtracted exponential is positive.

Answer: $(0, 1)$; $y = 7$; range $y < 7$.

(b) Find its x-intercept to three decimals. [3 marks]

$2^{x+1} = \frac{7}{3}$, so $x = \frac{\log(\frac{7}{3})}{\log 2} - 1$.

Answer: $(0.222, 0)$

5. An exponential model passes through $(2, 45)$ and $(5, 151.875)$. [4 marks]

(a) Find its one-unit multiplier. [2 marks]

$b^3 = \frac{151.875}{45} = 3.375$, so $b = 1.5$.

Answer: 1.5

(b) Find the initial value and the rule. [2 marks]

$A = \frac{45}{(1.5)^2} = 20$.

Answer: $A = 20;\ y = 20(1.5)^t$.

6. A 320 mg sample has a 6-hour half-life. [5 marks]

(a) Write a model with t in hours and find its mass after 18 hours. [2 marks]

18 hours is three half-lives; $\frac{320}{8} = 40$.

Answer: $M(t) = 320(\frac{1}{2})^{\frac{t}{6}}$; 40 mg.

(b) Find the time when 100 mg remains, to two decimals. [3 marks]

$t = \frac{6\log(\frac{100}{320})}{\log(\frac{1}{2})}$.

Answer: 10.07 hours.

7. Work with logarithmic statements. [4 marks]

(a) Write $\log_5 z = -2$ in exponential form and find z. [2 marks]

The log value is the exponent on base 5.

Answer: $z = 5^{-2} = \frac{1}{25}$

(b) Find the domain of $\log(8 - 2x)$. [2 marks]

$8 - 2x$ must be positive.

Answer: $x < 4$

8. Use log laws and check restrictions. [5 marks]

(a) Combine $3\log a + \log b - \log c$ for positive a, b, c. [2 marks]

Apply power, product and quotient laws.

Answer: $\log(\frac{a^3 b}{c})$

(b) Solve $\log(x+6) - \log x = \log 4$. [3 marks]

Domain $x > 0$; $\frac{x+6}{x} = 4$, giving $x = 2$.

Answer: $x = 2$.

9. An amount obeys $Q(t) = 90(0.92)^t$. [5 marks]

(a) Solve $Q(t) < 30$ for real t. [3 marks]

$90(0.92)^t < 30$ gives $(0.92)^t < \frac{1}{3}$; dividing by a negative log reverses the inequality.

Answer: $t > \frac{\log(\frac{1}{3})}{\log 0.92} \approx 13.176$

(b) If measured yearly, state the first completed year satisfying it. [2 marks]

Choose the first integer strictly greater than the real crossing time.

Answer: 14

10. Two proposed sales models are $L(t) = 100 + 20t$ and $E(t) = 100(1.15)^t$, with t in months. [5 marks]

(a) Compare their predictions after two months. [2 marks]

$L(2) = 140$ and $E(2) = 100(1.15)^2 = 132.25$.

Answer: $L = 140$; $E = 132.25$.

(b) Explain what extra evidence is needed to choose between the models. [3 marks]

Two plausible rules do not establish whether changes are additive or multiplicative.

Compare additional observations and consider capacity/seasonal effects.

Answer: Further time-matched observations and a plausible description of how sales change.