Exponential and logarithmic functions - Practice test 1

10 questions | 47 marks | 70 minutes

Build from exact powers and radicals to graph features, logarithmic equations and contextual model checks.

Equivalent correct methods earn the corresponding marks.

1. Evaluate and simplify. State any given restriction when using a variable. [4 marks]

(a) Evaluate $16^{\frac{3}{4}}$ and $25^{-\frac{1}{2}}$. [2 marks]

$\sqrt[4]{16}=2$ and $2^3=8$; $25^{-\frac{1}{2}}=\frac{1}{\sqrt{25}}=\frac{1}{5}$.

Answer: 8 and $\frac{1}{5}$.

(b) Simplify $\frac{(x^3)^2}{x^4}$ for $x\neq 0$. [2 marks]

The exponent is $3\times 2-4=2$.

Answer: x^2

2. Give exact answers. [5 marks]

(a) Simplify $\sqrt{75}-\sqrt{12}$. [2 marks]

$5\sqrt{3}-2\sqrt{3}=3\sqrt{3}$.

Answer: $3\sqrt{3}$

(b) Rationalise $\frac{4}{\sqrt{3}+1}$. [3 marks]

Multiply by the conjugate: $\frac{4(\sqrt{3}-1)}{3-1}=2(\sqrt{3}-1)$.

Answer: $2\sqrt{3}-2$

3. A material has mass 60 g, 72 g and 86.4 g at the ends of days 0, 1 and 2. [4 marks]

Observed mass

Time (days)	0	1	2
Mass (g)	60	72	86.4

(a) Show that an exponential model fits these values. [2 marks]

$\frac{72}{60} = 1.2$ and $\frac{86.4}{72} = 1.2$.

Answer: The common daily multiplier is 1.2.

(b) Write a model and predict the mass on day 3. [2 marks]

$M(3) = 60(1.2)^3 = 103.68$.

Answer: $M(t) = 60(1.2)^t$; 103.68 g.

4. Consider $f(x) = 2^{x-2} + 3$. [5 marks]

(a) State the transformations from $y = 2^x$ and the horizontal asymptote. [2 marks]

The inner subtraction moves right; the outer addition moves up.

Answer: Right 2 and up 3; asymptote $y = 3$.

(b) Find the y-intercept and explain whether there is an x-intercept. [3 marks]

$f(0) = 2^{-2} + 3 = 3.25$; all outputs exceed 3.

Answer: $(0, 3.25)$; no x-intercept.

5. A quantity is 50 at $t = 0$ and 98 at $t = 2$. Assume $y = Ab^t$ with b positive. [4 marks]

(a) Find A and b. [2 marks]

$A = 50$ and $b^2 = \frac{98}{50} = 1.96$; take the positive root.

Answer: $A = 50$, $b = 1.4$.

(b) Predict y at $t = 3$. [2 marks]

$50(1.4)^3 = 137.2$.

Answer: 137.2

6. 2000 dollars is invested at 5% yearly for 4 years. Ignore fees and tax. [5 marks]

(a) Find the compound-interest balance, rounded to cents. [2 marks]

$2000(1.05)^4 = 2431.0125$.

Answer: 2431.01 dollars.

(b) Compare it with simple interest at the same stated rate. [3 marks]

Simple balance $2000(1 + 0.05 \times 4) = 2400$.

Subtract 2400 from the unrounded compound balance, then round.

Answer: Simple balance 2400 dollars; compound is 31.01 dollars more.

7. Read each logarithm as an exponent. [5 marks]

(a) Evaluate $\log_4 64$. [2 marks]

$4^3 = 64$.

Answer: 3

(b) Find $\log 0.02$ to five decimal places and explain its sign. [3 marks]

$0.01 < 0.02 < 0.1$ so its common log lies between -2 and -1.

Answer: -1.69897; negative because the argument is between 0 and 1.

8. Solve $\log(x+1)+\log(x-1)=\log 8$. [5 marks]

(a) State the original domain and form a polynomial equation. [3 marks]

The arguments require $x+1>0$ and $x-1>0$.

Combine and equate arguments: $(x+1)(x-1)=8$.

Answer: $x>1$; $x^2-9=0$.

(b) Find and check the valid solution. [2 marks]

The candidates are 3 and -3; reject -3 because $x>1$.

At 3, the arguments are 4 and 2, whose product is 8.

Answer: $x=3$.

9. A model is $N(t)=6(1.1)^t$. [5 marks]

(a) Find the real time when $N=10$, to three decimals. [3 marks]

$t=\frac{\log(\frac{10}{6})}{\log 1.1}$.

Answer: $t=5.360$

(b) If t counts completed years, state the first year when N exceeds 10. [2 marks]

The continuous crossing is about 5.360; year 6 is the first larger integer.

Answer: Year 6.

10. A culture is observed for 0 to 5 hours. A model $N(t)=100(1.6)^t$ is fitted. The container holds at most 1500 cells. [5 marks]

(a) Use the model to predict the count at 8 hours. [2 marks]

$100(1.6)^8=4294.967296$; round to a whole count.

Answer: Approximately 4295 cells.

(b) Explain two reasons to question this prediction. [3 marks]

The fit was observed only through hour 5.

The unchanged exponential rule ignores the 1500-cell capacity.

Answer: It extrapolates beyond 5 hours and exceeds capacity.