Solving exponential equations with base-10 logs

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. Solve $8^x = 12$ to four decimal places. [2 marks]

$x = \frac{\log 12}{\log 8}$.

Answer: $x \approx 1.1950$

2. Solve $2^{x+3} = 7$ to four decimal places. [2 marks]

$x + 3 = \frac{\log 7}{\log 2}$, then subtract 3.

Answer: $x \approx -0.1926$

3. Solve $4(1.2)^t = 10$ to three decimal places. [2 marks]

Isolate $(1.2)^t = 2.5$; use $t = \frac{\log 2.5}{\log 1.2}$.

Answer: $t \approx 5.026$

4. A 600-unit quantity grows by 8% at the end of each year. In which completed year does it first exceed 1000? [3 marks]

$t > \frac{\log(\frac{1000}{600})}{\log 1.08} \approx 6.637$.

Year 7 is the first integer year; substitution verifies the threshold.

Answer: 7

5. Solve $(0.8)^t < 0.3$ for real t. [3 marks]

Take logs and divide by $\log 0.8 < 0$, reversing the inequality.

Answer: $t > \frac{\log 0.3}{\log 0.8} \approx 5.396$

6. Can $2^x = -3$ have a real solution? Explain. [2 marks]

2^x is positive for every real x, so it cannot equal a negative number.

Answer: No.