Marking guide | Trigonometry

Trigonometry - Applications and reasoning test

10 questions | 58 marks | 75 minutes

A more demanding paper requiring method selection, domain checks and interpretation. All necessary data are supplied.

Equivalent correct methods earn the corresponding marks.

1. A triangle is claimed to have $A = 60^\circ$, $a = 4$ cm and $b = 8$ cm. [6 marks]

(a) Show whether the data are possible. [3 marks]

$\sin B = \frac{8\sin 60^\circ}{4} = \sqrt{3} > 1.$

Answer: Impossible.

(b) If only b changes, find the greatest b compatible with the sine bound. State whether it gives a valid triangle. [3 marks]

$\frac{b\sin 60^\circ}{4} \le 1.$

At equality $B = 90^\circ$, leaving positive $C = 30^\circ$.

Answer: $b = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$ cm; valid with $B = 90^\circ$, $C = 30^\circ$.

2. Sides 4 cm and 6 cm enclose 150°. [6 marks]

(a) Find the third side and area exactly. [4 marks]

$c^2 = 16 + 36 - 48(-\frac{\sqrt{3}}{2}).$

$K = 12\sin 150^\circ = 6.$

Answer: $c = \sqrt{52 + 24\sqrt{3}}$ cm; $K = 6$ cm^2

(b) An included angle 30° gives the same area. Does it give the same third side? Explain. [2 marks]

Supplementary angles have equal sine but opposite cosine.

Answer: No; its squared third side is $52 - 24\sqrt{3}$.

3. A triangle has $A = 45^\circ$, $a = 7$ cm and $b = 8$ cm. [6 marks]

(a) Find all possible B values to one decimal place. [3 marks]

$\sin B = \frac{8\sin 45^\circ}{7} = \frac{4\sqrt{2}}{7}$.

$B \approx 53.9129^\circ$ or 126.0871°.

Answer: 53.9° or 126.1°.

(b) Find corresponding third angles and decide how many triangles exist. [3 marks]

$C = 135^\circ - B$; both are positive.

Answer: 81.1° or 8.9°; two triangles.

4. Two sectors each have full perimeter 20 cm. Sector I has angle 1 radian; sectorII has angle 2 radians. [6 marks]

(a) Find each radius and exact area. [4 marks]

Use $20 = r(\theta + 2)$.

Substitute each radius into $K = \frac{r^2\theta}{2}$.

Answer: I: $r = \frac{20}{3}$, $K = \frac{200}{9}$; II: $r = 5$, $K = 25$.

(b) Which has greater area and by how much? [2 marks]

$25 - \frac{200}{9} = \frac{25}{9}$.

Answer: Sector II by $\frac{25}{9}\ \text{cm}^2$.

5. A periodic displacement has minimum 2 cm, maximum 14 cm and period 10 s. At $t = 0$ it crosses the midline downwards. [6 marks]

(a) Write a sine rule consistent with the description. [3 marks]

Midline 8, amplitude 6, $B = \frac{\pi}{5}$; negative sine initially falls.

Answer: $s = 8 - 6\sin(\frac{\pi t}{5})$

(b) Find the first minimum and first return to the same downward crossing. [3 marks]

The first sine maximum occurs at argument $\frac{\pi}{2}$; one full cycle returns after 10 s.

Answer: Minimum at 2.5 s; downward crossing at 10 s.

6. Solve $\sin^2 x + \sin x = 0$. [5 marks]

(a) Find possible sine values. [2 marks]

$\sin x(\sin x + 1) = 0$.

Answer: 0 or -1.

(b) Find all roots on $0 \le x < 2\pi$. [3 marks]

Combine $\sin x = 0$ with $\sin x = -1$; exclude 2π.

Answer: $x = 0, \pi, \frac{3\pi}{2}$

7. Consider $\cos(3x - \frac{\pi}{2}) = 0$ for $0 \le x \le \pi$. [5 marks]

(a) Transform the x interval into an interval for the argument. [2 marks]

Multiply by 3, then subtract $\frac{\pi}{2}$.

Answer: $-\frac{\pi}{2} \le 3x - \frac{\pi}{2} \le \frac{5\pi}{2}$

(b) Find every solution. [3 marks]

Argument zeros are $-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$; solve for x.

Answer: $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$

8. An isosceles triangular panel has two equal sides 10 cm and included angle C. [6 marks]

(a) Find C values giving area 25 cm². [3 marks]

$25 = 50\sin C$, so $\sin C = \frac{1}{2}$.

Answer: 30° or 150°.

(b) Find the base length for each panel exactly and compare the shapes. [3 marks]

$b^2 = 200 - 200\cos C$.

Use $\cos 30^\circ = \frac{\sqrt{3}}{2}$ and $\cos 150^\circ = -\frac{\sqrt{3}}{2}$.

Answer: $\sqrt{200 - 100\sqrt{3}}$ cm or $\sqrt{200 + 100\sqrt{3}}$ cm; the obtuse panel has the longer base.

9. A proposed cosine model is $h = 5 + 2\cos(\frac{\pi t}{4})$. Measurements at $t = 0, 2, 4$ h are respectively 7,5,4 m. [6 marks]

(a) Identify which measurement disagrees with the model and give the predicted value. [3 marks]

$\cos \pi = -1$, so $h(4) = 3$.

Answer: The 4 h measurement; the model predicts 3 m.

(b) Explain two possible responses before using the model for a long prediction. [3 marks]

Do not discard a conflicting measurement merely to protect a chosen model.

Obtain more observations to decide whether the model is suitable.

Answer: Check measurement reliability and reconsider the fitted parameters or periodic assumptions.

10. A walker goes 100 m east from A to B, then 60 m on bearing 330° to C. [6 marks]

(a) Find C's east and north displacements from A exactly. [3 marks]

The second leg has east component $60\sin 330^\circ = -30$ and north component $60\cos 330^\circ = 30\sqrt{3}$.

Answer: East 70 m; north $30\sqrt{3}$ m.

(b) Find AC exactly and the bearing of C from A to one decimal place. [3 marks]

$AC = \sqrt{70^2 + (30\sqrt{3})^2} = 20\sqrt{19}$.

Bearing $= \tan^{-1}(\frac{70}{30\sqrt{3}}) \approx 53.4132^\circ$.

Answer: $20\sqrt{19}$ m ≈ 87.18 m; bearing 053.4°.