Trigonometry - Practice test 4

10 questions | 56 marks | 75 minutes

Interpret periodic models and combine triangle methods in practical situations.

Equivalent correct methods earn the corresponding marks.

1. An 8 m ramp makes 30° with horizontal ground. [5 marks]

(a) Find vertical rise and horizontal run exactly. [3 marks]

$\text{Rise} = 8\sin 30^\circ;\ \text{run} = 8\cos 30^\circ.$

Answer: 4 m and $4\sqrt{3}$ m

(b) A safety limit allows rise at most 3 m for this ramp length. Is the proposed ramp acceptable? [2 marks]

Its rise is 4 m, exceeding the limit by 1 m.

Answer: No.

2. Triangle data give $A = 40^\circ$, $a = 5$ cm and $b = 9$ cm. [5 marks]

(a) Use the sine rule to check whether any triangle exists. [3 marks]

$\sin B = \frac{9\sin 40^\circ}{5} \approx 1.1570 > 1.$

Answer: No triangle.

(b) Explain why trying the supplementary angle does not help. [2 marks]

The inconsistency occurs before inverse sine can give a real angle.

Answer: Neither angle can have sine greater than 1.

3. A triangle has sides 7 cm,8 cm and 9 cm. [6 marks]

(a) Find the cosine of the angle C opposite 9 cm. [3 marks]

$\cos C = \frac{49+64-81}{2\cdot 7\cdot 8} = \frac{2}{7}$.

Answer: $\frac{2}{7}$

(b) Use this to find its exact area. [3 marks]

$\sin C = \sqrt{1-\frac{4}{49}} = \frac{3\sqrt{5}}{7}$.

$K = 28(\frac{3\sqrt{5}}{7}) = 12\sqrt{5}$.

Answer: $12\sqrt{5}\ \text{cm}^2$

4. A sector has angle 3 radians and full perimeter 10 cm. [6 marks]

(a) Find radius and arc length. [3 marks]

$P = r\theta + 2r = 5r = 10$.

Answer: $r = 2$ cm; arc 6 cm.

(b) Find its area and describe why 3 radians is a valid sector angle. [3 marks]

$K = \frac{1}{2}(4)(3) = 6$.

Answer: 6 cm²; it is between 0 and 2π.

5. An angle θ is in quadrant IV and $\cos\theta = \frac{5}{13}$. [5 marks]

(a) Find sine and tangent exactly. [3 marks]

$\sin^2\theta = \frac{144}{169}$; choose negative sine.

Answer: $-\frac{12}{13}$, $-\frac{12}{5}$

(b) Find $\sin^2\theta + \cos^2\theta$ and explain its geometric meaning. [2 marks]

$\frac{144}{169} + \frac{25}{169} = 1$.

Answer: 1; squared coordinates sum to the squared unit radius.

6. A height model is $h = 6 - 4\cos(\frac{\pi t}{6})$ m, with t in seconds. [6 marks]

(a) Find range, period and starting value. [3 marks]

Read midline/amplitude; evaluate $t = 0$.

Answer: $[2, 10]$ m;12 s;2 m.

(b) Find $h = 8$ during the first cycle $0 \le t < 12$. [3 marks]

$\cos(\frac{\pi t}{6}) = -\frac{1}{2}$; arguments $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$.

Answer: $t = 4$ s and 8 s.

7. A student claims $\sin(2x) = 2\sin x$ for all x. [5 marks]

(a) Give an exact counterexample. [3 marks]

$\sin \pi = 0$, whereas $2\sin(\frac{\pi}{2}) = 2$.

Answer: $x = \frac{\pi}{2}$: left 0, right 2.

(b) Explain the difference between an equation and an identity. [2 marks]

One counterexample disproves a universal identity.

Answer: An equation can hold for selected x; an identity holds throughout its domain.

8. Consider $y = \tan(4x)$. [6 marks]

(a) State its period and closest asymptotes to 0. [3 marks]

$T = \frac{\pi}{4}$; solve $4x = \pm\frac{\pi}{2}$.

Answer: $\frac{\pi}{4}$; $x = \pm\frac{\pi}{8}$.

(b) Solve $y = 0$ on $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$. [3 marks]

$4x$ is an integer multiple of π.

Answer: $x = -\frac{\pi}{4}, 0, \frac{\pi}{4}$

9. A wheel has centre 15 m above ground, radius 12 m and period 48 s. A rider starts at the highest point. [6 marks]

(a) Write a height rule. [3 marks]

Midline 15; amplitude 12; $B = \frac{2\pi}{48}$.

Answer: $h = 15 + 12\cos(\frac{\pi t}{24})$

(b) Find the first two times $h = 9$ m. [3 marks]

$\cos(\frac{\pi t}{24}) = -\frac{1}{2}$; arguments $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$.

Answer: 16 s and 32 s.

10. Two observers are 20 m apart on level ground on the same side of a tower. The nearer angle of elevation is 60° and the farther is 30°. [6 marks]

(a) Find the nearer observer's distance from the base. [3 marks]

Let it be x. Then $h = x\sqrt{3} = \frac{x+20}{\sqrt{3}}$.

$3x = x + 20$, so $x = 10$.

Answer: 10 m.

(b) Find tower height exactly and state one modelling assumption. [3 marks]

$h = 10\tan 60^\circ = 10\sqrt{3}$.

The tower is vertical and the base/observer reference levels match.

Answer: $10\sqrt{3}$ m; observers at the same ground/eye reference level.