Trigonometry - Practice test 3

10 questions | 58 marks | 75 minutes

Connect representations and justify choices, including diagrams, tables and inverse problems.

Equivalent correct methods earn the corresponding marks.

1. A triangle has $A = 35^\circ$, $a = 12$ cm and $b = 8$ cm. [6 marks]

(a) Find B to one decimal place. [3 marks]

$\sin B = \frac{8\sin 35^\circ}{12} \approx 0.382384$.

$B \approx 22.4814^\circ$.

Answer: 22.5°.

(b) Explain why there is only one triangle, then find C. [3 marks]

$35^\circ + 157.5^\circ$ exceeds 180°; $C = 180^\circ - 35^\circ - B$.

Answer: The supplement 157.5° is invalid; $C \approx 122.5^\circ$.

2. Work with angle units. [5 marks]

(a) Convert 315° and -135° to radians. [2 marks]

Multiply each by $\frac{\pi}{180}$ and simplify.

Answer: $\frac{7\pi}{4}$ and $-\frac{3\pi}{4}$

(b) List all angles coterminal with $\frac{\pi}{4}$ in $-2\pi \le x \le 2\pi$. [3 marks]

$x = \frac{\pi}{4} + 2k\pi$; retain $k = -1, 0$.

Answer: $-\frac{7\pi}{4}, \frac{\pi}{4}$

3. A sector has radius r and angle 2 radians. Its area is 25 cm². [6 marks]

(a) Find r and its arc length. [3 marks]

$25 = \frac{r^2(2)}{2} = r^2$; select positive r.

Answer: $r = 5$ cm; arc 10 cm.

(b) Find the full boundary length and compare it with the arc. [3 marks]

$\text{Boundary} = 10 + 2 \times 5 = 20.$

Answer: 20 cm; twice the arc.

4. A triangle has sides 10 cm,11 cm and 17 cm. [6 marks]

(a) Find its largest angle to one decimal place. [3 marks]

$\cos C = \frac{100 + 121 - 289}{220} = -\frac{17}{55}.$

$C \approx 108.00445^\circ.$

Answer: $108.0^\circ.$

(b) Explain why the result must be obtuse without calculating inverse cosine. [3 marks]

Also $17^2 > 10^2 + 11^2$; the opposite angle exceeds 90°.

Answer: The cosine is negative.

5. A chord subtends 120° at the centre of a circle of radius 6 cm. [6 marks]

(a) Find chord length exactly. [3 marks]

Bisect the isosceles triangle; $\text{half} - \text{chord} = 6\sin 60^\circ.$

Answer: $6\sqrt{3}$ cm

(b) Find the minor arc and explain why it exceeds the chord. [3 marks]

$\text{Arc} = 6 \times \frac{2\pi}{3} = 4\pi \approx 12.57;\ \text{chord} \approx 10.39.$

A curved route is longer than the straight chord.

Answer: 4π cm

6. The table records height at equal time intervals during one smooth repeating cycle. [6 marks]

Height observations during one cycle

Time t (h)	Height h (m)
0	8
2	5
4	2
6	5
8	8

(a) Give a cosine rule consistent with the smooth cycle shown by these values. [3 marks]

Midline 5, amplitude 3, full cycle 8 h and starts at maximum.

Answer: $h = 5 + 3\cos(\frac{\pi t}{4})$

(b) State one assumption needed to predict h at 10 h, and give that prediction. [3 marks]

$\cos(\frac{5\pi}{2}) = 0$.

Answer: Assume the same cycle continues; $h(10) = 5$ m.

7. Let $g(x) = 4 - 2\cos(2x - \frac{\pi}{3})$. [6 marks]

(a) Find its amplitude, period and horizontal shift. [3 marks]

$2x - \frac{\pi}{3} = 2(x - \frac{\pi}{6})$.

Answer: $2; \pi; \frac{\pi}{6}$ to the right.

(b) Solve $g(x) = 3$ on $0 \le x < \pi$. [3 marks]

$\cos(2x - \frac{\pi}{3}) = \frac{1}{2}$.

The allowed arguments $-\frac{\pi}{3}$ and $\frac{\pi}{3}$ give the roots.

Answer: $x = 0, \frac{\pi}{3}$

8. Solve $2\sin^2 x - 3\sin x + 1 = 0$. [5 marks]

(a) Find possible sine values. [2 marks]

$(2u-1)(u-1)=0$, with $u=\sin x$.

Answer: $\sin x = \frac{1}{2}$ or 1.

(b) Find every root for $0 \le x < 2\pi$. [3 marks]

Use both positive-sine values and combine distinct roots.

Answer: $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

9. Two survey rays from O have bearings 020° and 140°. Points P,Q are respectively 30 m and 50 m along the rays. [6 marks]

(a) Find the interior angle POQ and distance PQ exactly. [3 marks]

Bearing difference 120°; $PQ^2 = 900 + 2500 - 3000\cos 120^\circ = 4900$.

Answer: $120^\circ;\ 70\ \text{m}$

(b) Find the area OPQ exactly. [3 marks]

$K = \frac{1}{2}(30)(50)\sin 120^\circ$.

Answer: $375\sqrt{3}\ \text{m}^2$

10. A triangular frame uses sides 6 m and 10 m with a flexible joint. [6 marks]

(a) Find the greatest possible enclosed area and corresponding angle. [3 marks]

$\text{Area} = 30\sin C$, whose maximum is 30.

Answer: 30 m² at 90°.

(b) Find both included angles giving area 15 m², and explain why the frames have different third sides. [3 marks]

$\sin C = \frac{1}{2}$.

Cosine rule contains $\cos C$, which changes sign for supplementary angles.

Answer: 30° and 150°; their cosines differ.