Trigonometry - Practice test 2

10 questions | 60 marks | 75 minutes

Use exact circular measures and explain why solutions are accepted or rejected.

Equivalent correct methods earn the corresponding marks.

1. A right triangle has hypotenuse 25 cm and one leg 7 cm. [6 marks]

(a) Find the other leg and the angle opposite 7 cm to one decimal place. [3 marks]

$a=\sqrt{625-49}=24;\ \theta=\sin^{-1}\left(\frac{7}{25}\right)\approx 16.2602^\circ.$

Answer: 24 cm; 16.3°.

(b) State the three ratios for that angle. [3 marks]

Use opposite 7, adjacent 24, hypotenuse 25.

Answer: $\frac{7}{25}, \frac{24}{25}, \frac{7}{24}$

2. A triangle has sides 9 cm and 13 cm with included angle 50°. [6 marks]

(a) Find the third side to two decimal places. [3 marks]

$c=\sqrt{81+169-234\cos 50^\circ}\approx 9.97936.$

Answer: 9.98 cm.

(b) Find its area to two decimal places. [3 marks]

$K=58.5\sin 50^\circ\approx 44.8136.$

Answer: 44.81 cm².

3. Triangle ABC has $A = 25^\circ$, $a = 6$ cm and $b = 9$ cm. [6 marks]

(a) Find every possible B to one decimal place. [3 marks]

$\sin B = 1.5 \sin 25^\circ \approx 0.633927.$

$B \approx 39.3405^\circ$ or 140.6596°.

Answer: 39.3° or 140.7°.

(b) Find corresponding C values. [3 marks]

Subtract each B and 25° from 180°.

Answer: 115.7° or 14.3°.

4. A triangle has sides 5 cm and 12 cm enclosing an obtuse angle C. Its area is 15 cm². [6 marks]

(a) Find C. [3 marks]

$15 = 30 \sin C$, so $\sin C = \frac{1}{2}$; choose the obtuse angle.

Answer: 150°.

(b) Find the third side exactly. [3 marks]

$c^2 = 25 + 144 - 120 \cos 150^\circ = 169 + 60\sqrt{3}$.

Answer: $\sqrt{169 + 60\sqrt{3}}$ cm

5. A sector has angle $\frac{5\pi}{6}$ and arc length 10π cm. [6 marks]

(a) Find its radius and area exactly. [4 marks]

$r = \frac{10\pi}{\frac{5\pi}{6}} = 12.$

$K = \frac{144(\frac{5\pi}{6})}{2} = 60\pi.$

Answer: $r = 12 \text{ cm};\ K = 60\pi \text{ cm}^2$

(b) Find the area of the complementary major sector. [2 marks]

Whole area 144π minus 60π.

Answer: $84\pi \text{ cm}^2$

6. Use the unit circle. [6 marks]

(a) Find $\cos(\frac{7\pi}{6})$ and $\tan(\frac{3\pi}{4})$ exactly. [3 marks]

Apply quadrant III andII signs respectively.

Answer: $-\frac{\sqrt{3}}{2}$ and -1

(b) Given $\sin\theta = \frac{12}{13}$ in quadrant II, find $\cos\theta$ and $\tan\theta$. [3 marks]

$\cos^2\theta = 1 - \frac{144}{169} = \frac{25}{169}$; choose negative cosine.

Answer: $-\frac{5}{13}, -\frac{12}{5}$

7. Consider $f(x) = -1 + 2\sin(3x)$. [6 marks]

(a) State amplitude, midline and period. [3 marks]

Use $|A|, D$ and $\frac{2\pi}{|B|}$.

Answer: 2; $y = -1; \frac{2\pi}{3}$.

(b) Find all maxima for $0 \le x < 2\pi$. [3 marks]

$3x = \frac{\pi}{2} + 2k\pi$; retain x in the interval.

Answer: $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

8. Solve each equation. [6 marks]

(a) $\tan(2x) = 1$ for $0 \le x < \pi$. [3 marks]

$2x = \frac{\pi}{4} + k\pi$.

Answer: $\frac{\pi}{8}, \frac{5\pi}{8}$

(b) $2\cos^2 x - 1 = 0$ for $0 \le x < 2\pi$. [3 marks]

$\cos x = \pm\frac{\sqrt{2}}{2}$; use all four quadrants.

Answer: $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

9. A rotating arm's endpoint has height $h = 9 + 4\sin(\frac{\pi t}{5})$ metres, t in seconds. [6 marks]

(a) Find its range, period and first maximum time. [3 marks]

Amplitude 4, midline 9; argument $\frac{\pi}{2}$ gives the first maximum.

Answer: $[5, 13]$ m;10 s;2.5 s.

(b) Find the first positive time when $h = 7$. [3 marks]

$\sin(\frac{\pi t}{5}) = -\frac{1}{2}$; first positive argument $\frac{7\pi}{6}$.

Answer: $\frac{35}{6}$ s ≈ 5.83 s.

10. A circular running track has radius 30 m. A runner travels at 3 m/s. [6 marks]

(a) How many radians does the runner turn through in 20 s? [3 marks]

Arc distance 60 m; angle $\frac{60}{30} = 2$.

Answer: 2 rad.

(b) Find the time for one full lap and explain the unit conversion. [3 marks]

Circumference 60π m; divide by 3 m/s.

Answer: 20π s ≈ 62.83 s