Marking guide | Trigonometry

Trigonometry - Practice test 1

10 questions | 59 marks | 75 minutes

A balanced paper from triangle ratios to periodic applications. Give exact values where requested and retain precision in calculations.

Equivalent correct methods earn the corresponding marks.

1. A right triangle has legs 5 cm and 12 cm. θ is opposite 5 cm. [5 marks]

(a) Find its hypotenuse and all three ratios. [3 marks]

$h = \sqrt{25 + 144} = 13.$

Apply the three ratio definitions.

Answer: $h = 13;\ \sin\theta = \frac{5}{13}, \cos\theta = \frac{12}{13}, \tan\theta = \frac{5}{12}.$

(b) Find θ to one decimal place. [2 marks]

$\theta = \tan^{-1}\left(\frac{5}{12}\right) \approx 22.6199^\circ.$

Answer: $22.6^\circ.$

2. In triangle ABC, $A = 45^\circ$, $B = 75^\circ$ and $a = 6$ cm. You may use $\sin 75^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}$. [6 marks]

(a) Find C and b exactly. [3 marks]

$C = 180 - 45 - 75 = 60^\circ.$

$b = \frac{6\sin 75^\circ}{\sin 45^\circ} = 3(\sqrt{3} + 1).$

Answer: $C = 60^\circ;\ b = 3(\sqrt{3} + 1)$ cm.

(b) Find c exactly. [3 marks]

$c = \frac{6\sin 60^\circ}{\sin 45^\circ} = 3\sqrt{6}.$

Answer: $3\sqrt{6}$ cm.

3. Given $A = 30^\circ$, $a = 10$ cm and $b = 10\sqrt{3}$ cm. [6 marks]

(a) Find every possible angle B. [3 marks]

$\sin B = \frac{b\sin A}{a} = \frac{\sqrt{3}}{2}$.

Answer: 60° or 120°.

(b) Find the corresponding C values and justify the number of triangles. [3 marks]

Both choices leave a positive third angle.

Answer: $C = 90^\circ$ or 30°; two triangles.

4. Sides 4 cm and 9 cm enclose 60°. [6 marks]

(a) Find the third side exactly. [3 marks]

$c^2 = 16 + 81 - 72(\frac{1}{2}) = 61$.

Answer: $\sqrt{61}$ cm

(b) Find the area exactly. [3 marks]

$K = 18\sin 60^\circ = 9\sqrt{3}$.

Answer: $9\sqrt{3}$ cm^2

5. A sector has radius 9 cm and angle 80°. [6 marks]

(a) Find its arc length and area exactly. [4 marks]

$80^\circ = \frac{4\pi}{9}$; substitute in both radian formulas.

Answer: $s = 4\pi$ cm; $K = 18\pi$ cm^2.

(b) Find its full perimeter. [2 marks]

Add the two straight radii to the arc.

Answer: $18 + 4\pi$ cm

6. Work with exact trigonometric values. [6 marks]

(a) Find sine, cosine and tangent of 240°. [3 marks]

Reference angle 60° in quadrant III.

Answer: $-\frac{\sqrt{3}}{2}, -\frac{1}{2}, \sqrt{3}$

(b) Solve $\sin x = -\frac{\sqrt{2}}{2}$ on $0 \le x < 2\pi$. [3 marks]

Negative sine occurs in quadrantsIII andIV.

Answer: $\frac{5\pi}{4}, \frac{7\pi}{4}$

7. Let $y = 2 + 3\cos(\frac{x}{2})$. [6 marks]

(a) State the range, period and value at 0. [3 marks]

Midline 2, amplitude 3, $B = \frac{1}{2}$.

Answer: Range $[-1, 5]$; period 4π; $y(0) = 5$.

(b) Find $y = 2$ for $0 \le x < 4\pi$. [3 marks]

$\cos(\frac{x}{2}) = 0$; solve over $0 \le \frac{x}{2} < 2\pi$.

Answer: $x = \pi, 3\pi$

8. Solve each equation over its stated interval. [6 marks]

(a) $2\sin x = 1$, $-\pi \le x \le \pi$. [3 marks]

Divide by 2; use positive sine.

Answer: $\frac{\pi}{6}, \frac{5\pi}{6}$

(b) $\cos(2x) = -1$, $0 \le x < 2\pi$. [3 marks]

$2x = \pi$ or 3π in the doubled interval.

Answer: $\frac{\pi}{2}, \frac{3\pi}{2}$

9. A tide varies between 1.5 m and 4.5 m with period 12 h. It is at its maximum at $t = 0$. [6 marks]

(a) Write a cosine model. [3 marks]

Midline 3, amplitude 1.5, $B = \frac{\pi}{6}$.

Answer: $d = 3 + 1.5\cos(\frac{\pi t}{6})$

(b) Find the first two nonnegative times at mean depth. [3 marks]

Set cosine to 0 and interpret the two crossings.

Answer: 3 h and 9 h.

10. Two paths from a camp have lengths 40 m and 55 m with angle 120° between them. [6 marks]

(a) Find the distance between their endpoints, to two decimal places. [3 marks]

$d^2 = 1600 + 3025 - 4400(-\frac{1}{2}) = 6825.$

Answer: $5\sqrt{273}\ \text{m} \approx 82.61\ \text{m}$

(b) Find the triangle’s area exactly. [3 marks]

$K = \frac{1}{2}(40)(55)\sin 120^\circ.$

Answer: $550\sqrt{3}\ \text{m}^2$