Quadratics and polynomial graphs - Challenge and applications

Name: _____ Date: _____

10 questions | 54 marks | 75 minutes

A demanding mixed paper combining parameter reasoning, polynomial structure and constrained decisions.

A scientific calculator may be used.

1. Compare $A(x) = (x+3)^2$ and $B(x) = x^2 + 9$. [4 marks]

(a) Find every x for which $A(x) = B(x)$. [2 marks]

(b) Explain why one example of equality does not prove these expressions are identical. [2 marks]

2. A quadratic has vertex $(2, -3)$ and passes through $(-1, 15)$. [5 marks]

(a) Find the rule in vertex and expanded forms. [3 marks]

(b) Find the exact real roots. [2 marks]

3. Consider $x^2 - 4x = 0$ and $x^2 - 4 = 0$. [4 marks]

(a) Solve both equations and explain why their solution sets differ. [2 marks]

(b) A student divides the first equation by x. Explain precisely what information is lost. [2 marks]

4. The equation $ax^2 + 4x + 1 = 0$ is to have two distinct positive roots. [6 marks]

(a) Use the root sum and product to show no such real a exists. [3 marks]

(b) Find the values of a for which it has two distinct negative real roots. [3 marks]

5. A monic cubic $p(x) = x^3 + ax^2 + bx - 8$ has factors $x - 2$ and $x + 1$. [6 marks]

(a) Find a and b. [4 marks]

(b) Find the remaining factor and all roots. [2 marks]

6. Consider $g(x) = x^4 - kx^2 + 4$. [6 marks]

(a) Find k for which the quadratic in $u = x^2$ has a repeated root. [2 marks]

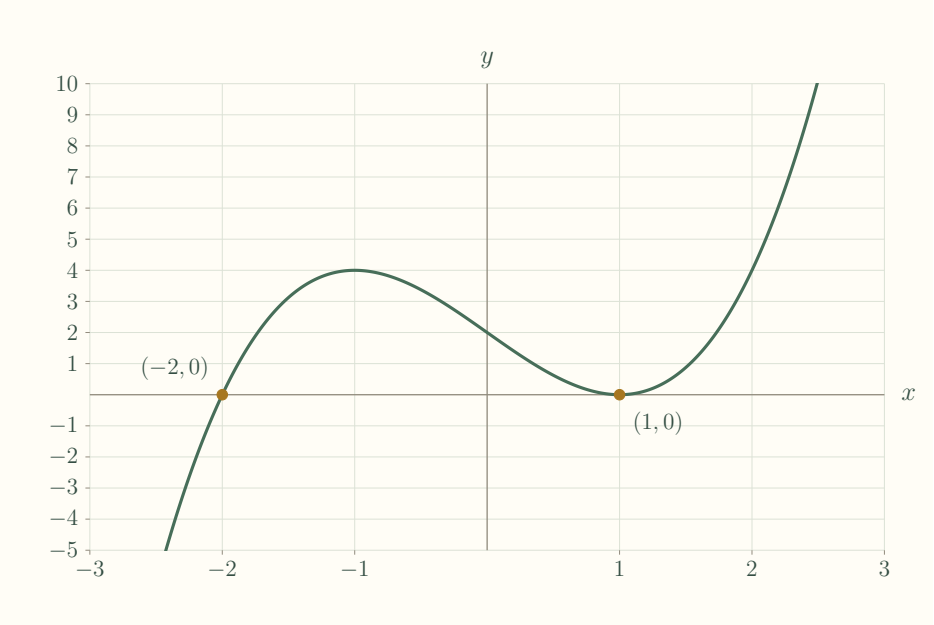
(b) For each value, state every real x-root and its multiplicity. [4 marks]

7. The rule $f(x) = -(x-5)^2 + 9$ is restricted to $1 \leq x \leq 4$. [4 marks]

(a) Find its range on that domain. [3 marks]

(b) Explain why 9 is not an attained output on this domain. [1 mark]

8. The graph shown has rule $p(x) = (x+2)(x-1)^2$. [6 marks]



(a) Explain its behaviour at each x-intercept and state $p(0)$. [3 marks]

(b) Find every intersection with the horizontal line $y = 2$ exactly. [3 marks]

9. Two positive numbers differ by 2 and their product is 35. [6 marks]

(a) Find the numbers using a quadratic equation. [3 marks]

(b) Form a quadratic equation with integer coefficients whose roots are the reciprocals of these numbers. [3 marks]

10. A seller models the number of items sold as $N = 26 - 2p$, where p is a whole-dollar price with $0 < p < 13$. Fixed daily costs are 60 dollars. [7 marks]

(a) Write profit as a quadratic and find every allowed price giving positive profit. [4 marks]

(b) Find the best allowed price or prices and the predicted profit. [3 marks]