Marking guide | Quadratics and polynomial graphs

Quadratics and polynomial graphs - Challenge and applications

10 questions | 54 marks | 75 minutes

A demanding mixed paper combining parameter reasoning, polynomial structure and constrained decisions.

Equivalent correct methods earn the corresponding marks.

1. Compare $A(x) = (x+3)^2$ and $B(x) = x^2 + 9$. [4 marks]

(a) Find every x for which $A(x) = B(x)$. [2 marks]

Expand A as $x^2 + 6x + 9$.

Equality reduces to $6x = 0$.

Answer: $x = 0$

(b) Explain why one example of equality does not prove these expressions are identical. [2 marks]

For example at $x = 1$, $A = 16$ but $B = 10$.

Answer: An identity must hold for every allowed x; these agree only at zero.

2. A quadratic has vertex $(2, -3)$ and passes through $(-1, 15)$. [5 marks]

(a) Find the rule in vertex and expanded forms. [3 marks]

Substitute $(-1, 15)$: $15 = 9a - 3$, so $a = 2$.

Expand the square.

Answer: $y = 2(x-2)^2 - 3 = 2x^2 - 8x + 5$.

(b) Find the exact real roots. [2 marks]

Set $(x-2)^2 = \frac{3}{2}$.

Take both square-root branches.

Answer: $x = 2 \pm \frac{\sqrt{6}}{2}$

3. Consider $x^2 - 4x = 0$ and $x^2 - 4 = 0$. [4 marks]

(a) Solve both equations and explain why their solution sets differ. [2 marks]

The first factors as $x(x-4)$.

The second factors as $(x-2)(x+2)$.

Answer: First: 0,4; second: -2,2.

(b) A student divides the first equation by x. Explain precisely what information is lost. [2 marks]

Division by x requires x nonzero.

The original equation is satisfied when $x = 0$.

Answer: The solution $x = 0$.

4. The equation $ax^2 + 4x + 1 = 0$ is to have two distinct positive roots. [6 marks]

(a) Use the root sum and product to show no such real a exists. [3 marks]

If $a = 0$ the equation is linear and has only one root, so assume a is nonzero.

Positive root product requires $\frac{1}{a} > 0$, hence $a > 0$.

But then root sum $\frac{-4}{a}$ is negative, so the two roots cannot both be positive.

Answer: No real a gives two positive roots.

(b) Find the values of a for which it has two distinct negative real roots. [3 marks]

For two negative roots, positive product and negative sum require $a > 0$.

Two distinct real roots require $16 - 4a > 0$.

Combine the conditions.

Answer: $0 < a < 4$.

5. A monic cubic $p(x) = x^3 + ax^2 + bx - 8$ has factors $x - 2$ and $x + 1$. [6 marks]

(a) Find a and b. [4 marks]

$p(2) = 0$ gives $2a + b = 0$.

$p(-1) = 0$ gives $a - b = 9$.

Solve the two equations.

Answer: $a = 3$, $b = -6$.

(b) Find the remaining factor and all roots. [2 marks]

The full factorisation is $(x - 2)(x + 1)(x + 4)$.

Expansion confirms the coefficients and constant.

Answer: Remaining factor $x + 4$; roots -4,-1,2.

6. Consider $g(x) = x^4 - kx^2 + 4$. [6 marks]

(a) Find k for which the quadratic in $u = x^2$ has a repeated root. [2 marks]

The u-equation is $u^2 - ku + 4 = 0$.

Set its discriminant $k^2 - 16 = 0$.

Answer: $k = 4$ or $k = -4$.

(b) For each value, state every real x-root and its multiplicity. [4 marks]

At $k = 4$, $g = (x^2 - 2)^2$.

At $k = -4$, $g = (x^2 + 2)^2$, which is strictly positive for real x.

Answer: $k = 4$: $x = \pm\sqrt{2}$, each double. $k = -4$: no real roots.

7. The rule $f(x) = -(x - 5)^2 + 9$ is restricted to $1 \le x \le 4$. [4 marks]

(a) Find its range on that domain. [3 marks]

The function is increasing to the left of $x = 5$.

Evaluate $f(1) = -7$ and $f(4) = 8$; endpoints are included.

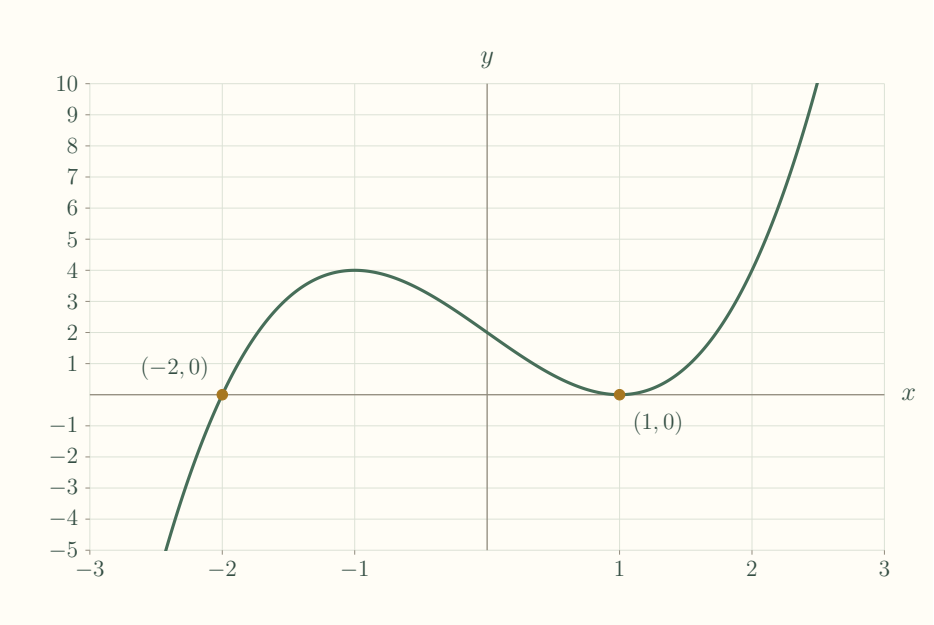
Answer: $-7 \le y \le 8$.

(b) Explain why 9 is not an attained output on this domain. [1 mark]

The unrestricted vertex is not part of the restricted graph.

Answer: It requires $x = 5$, which is excluded.

8. The graph shown has rule $p(x) = (x+2)(x-1)^2$. [6 marks]



(a) Explain its behaviour at each x-intercept and state $p(0)$. [3 marks]

Use odd/even factor multiplicities.

Substitute zero.

Answer: Crosses at -2; touches at 1; $p(0) = 2$.

(b) Find every intersection with the horizontal line $y = 2$ exactly. [3 marks]

Expand p as $x^3 - 3x + 2$.

Set $p = 2$, so $x(x^2 - 3) = 0$.

Solve all factors.

Answer: $x = -\sqrt{3}, 0, \sqrt{3}$; all points have $y = 2$.

9. Two positive numbers differ by 2 and their product is 35. [6 marks]

(a) Find the numbers using a quadratic equation. [3 marks]

Let the smaller be $x > 0$: $x(x+2) = 35$.

Factor $x^2 + 2x - 35 = (x+7)(x-5)$.

Reject $x = -7$.

Answer: 5 and 7.

(b) Form a quadratic equation with integer coefficients whose roots are the reciprocals of these numbers. [3 marks]

The reciprocal sum is $\frac{1}{5} + \frac{1}{7} = \frac{12}{35}$.

The reciprocal product is $\frac{1}{35}$; multiply the monic equation by 35.

Answer: $35t^2 - 12t + 1 = 0$.

10. A seller models the number of items sold as $N = 26 - 2p$, where p is a whole-dollar price with $0 < p < 13$. Fixed daily costs are 60 dollars. [7 marks]

(a) Write profit as a quadratic and find every allowed price giving positive profit. [4 marks]

Profit is revenue pN minus 60.

Factor $P = -2(p-3)(p-10)$.

Positive profit requires $3 < p < 10$; retain whole-dollar values.

Answer: $P = -2p^2 + 26p - 60$; prices $4, 5, 6, 7, 8, 9$.

(b) Find the best allowed price or prices and the predicted profit. [3 marks]

The real-valued vertex is $p = 6.5$, between two allowed integers.

Check $p = 6$ and 7; symmetry gives the same value.

Values farther from the vertex give less profit.

Answer: $p = 6$ or $p = 7$, each giving 24 dollars profit.