Quadratics and polynomial graphs - Practice 4

10 questions | 49 marks | 65 minutes

Graphs, geometric dimensions and careful interpretation of restricted models.

Equivalent correct methods earn the corresponding marks.

1. Simplify and factorise. [4 marks]

(a) Expand $(3x+2)(x-4)$. [2 marks]

Distribute and collect $-12x+2x$.

Answer: $3x^2-10x-8$

(b) Factorise $5x^2-20x+15$. [2 marks]

Take out 5.

Factor x^2-4x+3.

Answer: $5(x-1)(x-3)$

2. For $9x^2+kx+4=0$: [4 marks]

(a) Find k for a repeated real root. [2 marks]

Set $k^2-4(9)(4)=0$.

Solve $k^2=144$.

Answer: $k=-12$ or 12.

(b) Give the repeated root for each k. [2 marks]

Use $x=\frac{-b}{2a}=\frac{-k}{18}$.

Answer: $k=12$ gives $x=\frac{-2}{3}$; $k=-12$ gives $x=\frac{2}{3}$.

3. Let $f(x) = -2x^2 + 8x + 3$. [5 marks]

(a) Find its vertex and maximum. [2 marks]

Complete $f = -2(x-2)^2 + 11$.

Answer: Vertex $(2, 11)$; maximum 11.

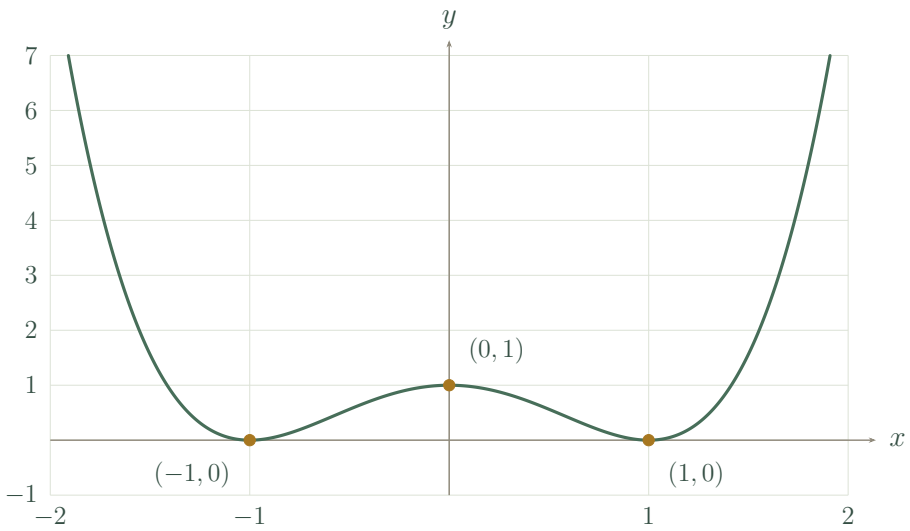
(b) Find both exact zeros. [3 marks]

Set $(x-2)^2 = \frac{11}{2}$.

Take roots and simplify $\sqrt{\frac{11}{2}} = \frac{\sqrt{22}}{2}$.

Answer: $x = 2 \pm \frac{\sqrt{22}}{2}$

4. The supplied graph is $y = (x^2 - 1)^2$. [6 marks]



(a) State its real zeros, their multiplicities and its range. [3 marks]

Factor as $(x-1)^2(x+1)^2$.

A square is nonnegative and reaches zero.

Answer: Zeros -1,1, each double; range $y \geq 0$.

(b) Find all x where $y = 1$. [3 marks]

Solve $(x^2 - 1)^2 = 1$, so $x^2 - 1 = \pm 1$.

This gives $x^2 = 0$ or 2.

Answer: $x = 0, \pm\sqrt{2}$

5. A quadratic has roots 0 and 6 and maximum value 18. [5 marks]

(a) Find its rule. [3 marks]

The axis is $x = 3$.

Substitute $(3, 18)$ into $y = ax(x - 6)$, giving $a = -2$.

Answer: $y = -2x(x - 6)$

(b) Find the two x-values where $y = 10$. [2 marks]

Solve $-2x^2 + 12x = 10$.

Divide by -2 to get $(x - 1)(x - 5) = 0$.

Answer: $x = 1, 5$

6. An 8 m by 12 m rectangular garden is surrounded by a path of uniform width w metres. The path alone has area 96 square metres. [5 marks]

(a) Form an equation for w. [3 marks]

Add twice the width to each garden dimension.

Subtract garden area from the outer area.

Answer: $(8 + 2w)(12 + 2w) - 96 = 96$, or $w^2 + 10w - 24 = 0$.

(b) Find the path width and outer dimensions. [2 marks]

Factor $(w + 12)(w - 2) = 0$.

Reject negative width -12.

Answer: $w = 2$ m; outer dimensions 12 m by 16 m.

7. A line $y = mx - 1$ is tangent to the parabola $y = x^2 - 4x + 3$. [5 marks]

(a) Write the equation whose roots are the intersection x-values. [2 marks]

Set the two y-rules equal and rearrange.

Answer: $x^2 - (4 + m)x + 4 = 0$

(b) Find both possible values of m. [3 marks]

Tangency requires discriminant $(4 + m)^2 - 16 = 0$.

Hence $4 + m = 4$ or -4.

Answer: $m = 0$ or $m = -8$.

8. Let $p(x) = 2x^3 - 3x^2 - 8x + 12$. [5 marks]

(a) Factorise by grouping. [3 marks]

Group $x^2(2x - 3) - 4(2x - 3)$.

Factor the difference of squares.

Answer: $(2x - 3)(x - 2)(x + 2)$

(b) State every root and the y-intercept of the graph. [2 marks]

Set each factor to zero.

Substitute $x = 0$.

Answer: Roots $-2, \frac{3}{2}, 2$; y-intercept $(0, 12)$.

9. Consider $f(x) = -(x + 1)^2(x - 2)(x - 4)$. [5 marks]

(a) Describe how the graph meets the x-axis and its end behaviour. [3 marks]

Multiplicities are two, one and one.

The leading term is $-x^4$.

Answer: Touches at -1; crosses at 2 and 4; both ends fall.

(b) Find $f(0)$ and explain its sign using the factors. [2 marks]

At zero the product is $-(1)(-2)(-4) = -8$.

The two negative linear factors give a positive product before the leading minus.

Answer: $f(0) = -8$.

10. A rectangle has perimeter 24 cm. Its width x is constrained by $0 < x \leq 4$. [5 marks]

(a) Write area in vertex form. [2 marks]

The length is $12 - x$.

Complete the square.

Answer: $A = x(12 - x) = -(x - 6)^2 + 36$.

(b) Find the greatest allowed area and explain why it is not 36. [3 marks]

The unrestricted vertex $x = 6$ is outside the allowed interval.

For $0 < x \leq 4$ the area increases, reaching $4(8) = 32$ at the included endpoint.

Answer: 32 square centimetres at $x = 4$ cm.