Quadratics and polynomial graphs - Practice 3

10 questions | 49 marks | 65 minutes

Tables, parameter conditions, reciprocal roots and modelling from measured data.

Equivalent correct methods earn the corresponding marks.

1. The table gives values of one quadratic function. [5 marks]

Values of y at selected x-values

x	-1	0	1	2	3
y	0	-6	-8	-6	0

(a) Use the symmetry to state the turning point and roots. [2 marks]

Equal values occur equally far from $x = 1$.

The table shows zeros at -1 and 3.

Answer: Vertex $(1, -8)$; roots -1 and 3.

(b) Find a rule for the function. [3 marks]

Use the two roots in factored form.

At $x = 0$, $-6 = -3a$, so $a = 2$.

Answer: $y = 2(x + 1)(x - 3)$

2. Solve the two equations. [4 marks]

(a) $6x^2 - x - 2 = 0$. [2 marks]

Factor $(3x - 2)(2x + 1) = 0$.

Answer: $x = \frac{2}{3}, \frac{-1}{2}$

(b) $x^2 - 3 = 2x$. [2 marks]

Rearrange $x^2 - 2x - 3 = (x - 3)(x + 1) = 0$.

Answer: $x = 3, -1$

3. For $f(x) = 2x^2 + 12x + k$: [5 marks]

(a) Write f in vertex form. [2 marks]

Factor 2 from the first two terms and complete the square.

Answer: $f(x) = 2(x+3)^2 + k - 18$

(b) Find k if the minimum is 5, then explain why there are no real roots. [3 marks]

Set $k - 18 = 5$.

A function with positive minimum cannot equal zero.

Answer: $k = 23$; all function values are at least 5.

4. The nonzero roots of $x^2 - 5x + 2 = 0$ are r and s. [5 marks]

(a) Find $\frac{1}{r} + \frac{1}{s}$ and $\frac{1}{rs}$. [3 marks]

Use $r + s = 5$ and $rs = 2$.

$\frac{1}{r} + \frac{1}{s} = \frac{r+s}{rs}$.

Answer: $\frac{5}{2}$ and $\frac{1}{2}$.

(b) Form a quadratic with roots $\frac{1}{r}$ and $\frac{1}{s}$ and integer coefficients. [2 marks]

Use the new sum and product in a monic equation.

Multiply by 2.

Answer: $2x^2 - 5x + 1 = 0$

5. Consider $kx^2 - 4x + 1 = 0$. [4 marks]

(a) For nonzero k, find the condition for two distinct real roots. [3 marks]

Require discriminant $16 - 4k > 0$.

Retain the stated nonzero-leading-coefficient condition.

Answer: $k < 4$ and $k \neq 0$.

(b) Solve separately when $k = 0$. [1 mark]

The equation becomes $-4x + 1 = 0$.

Answer: $x = \frac{1}{4}$

6. A family of horizontal lines $y = k$ meets $y = (x+1)^2 + 2$. [4 marks]

(a) Give the number of intersections for $k < 2$, $k = 2$ and $k > 2$. [2 marks]

The parabola has minimum 2 at $x = -1$.

Answer: Zero, one, and two respectively.

(b) Find the intersection coordinates when $k = 11$. [2 marks]

Solve $(x+1)^2 = 9$.

Use both square-root branches.

Answer: $(-4, 11)$ and $(2, 11)$.

7. A monic cubic $p(x) = x^3 + ax^2 + bx + 6$ has roots 1 and -2. [6 marks]

(a) Form and solve simultaneous equations for a and b. [4 marks]

$p(1) = 0$ gives $a + b = -7$.

$p(-2) = 0$ gives $2a - b = 1$.

Adding gives $3a = -6$.

Answer: $a = -2$, $b = -5$.

(b) Find the third root. [2 marks]

At $a = -2, b = -5$, divide p by $(x-1)(x+2) = x^2 + x - 2$.

The remaining factor is $x - 3$.

Answer: 3

8. Let $g(x) = x^4 - 2x^2 - 3$. [5 marks]

(a) Find every real root exactly. [3 marks]

Set $u = x^2$: $(u-3)(u+1) = 0$.

Reject $u = -1$ for real x.

Answer: $x = \pm\sqrt{3}$

(b) Explain why the equation has fewer than four real roots. [2 marks]

A quartic can have fewer than four real roots.

The two positive/negative square-root branches of $u = 3$ remain.

Answer: Two roots would require $x^2 = -1$ and are not real.

9. An arch meets the ground at $x = 0$ m and $x = 10$ m. Its height at $x = 2$ m is 8 m. [5 marks]

(a) Find a quadratic rule and state the physical domain. [3 marks]

Use $h = ax(x - 10)$.

Substitute $(2, 8)$ to get $a = -0.5$.

Answer: $h = -0.5x(x - 10)$, $0 \le x \le 10$.

(b) Find its maximum height and position. [2 marks]

The axis is halfway between the supports.

Substitute $x = 5$.

Answer: 12.5 m at $x = 5$ m.

10. A small event estimates attendance as $N = 120 - 4p$, where p is the ticket price in dollars and $0 < p < 30$. Revenue is pN dollars. [6 marks]

(a) Find the price giving greatest revenue and the predicted attendance. [3 marks]

Revenue $R = 120p - 4p^2 = -4(p - 15)^2 + 900$.

The vertex lies in the domain.

Answer: $p = 15$; attendance 60; revenue 900 dollars.

(b) Find both ticket prices predicted to produce 800 dollars revenue. [3 marks]

Solve $120p - 4p^2 = 800$.

Divide by -4 and factor $(p - 10)(p - 20) = 0$.

Both values lie in the allowed interval.

Answer: $p = 10$ or $p = 20$.