Quadratics and polynomial graphs - Practice 2

10 questions | 49 marks | 65 minutes

Error analysis, reconstruction, intersections and a box-volume application.

Equivalent correct methods earn the corresponding marks.

1. A student writes $(x-4)^2 = x^2 + 16$. [4 marks]

(a) Explain the error and give the correct expansion. [2 marks]

Expand $(x-4)(x-4)$.

Answer: $x^2 - 8x + 16$; the two cross-products were omitted.

(b) Factorise $4x^2 - 12x + 9$. [2 marks]

Recognise a perfect square with middle term $-12x$.

Answer: $(2x-3)^2$

2. A quadratic has roots -3 and 5 and passes through $(1, -16)$. [5 marks]

(a) Find a rule in factored form. [3 marks]

Use $y = a(x+3)(x-5)$.

At $(1, -16)$, $-16 = -16a$ so $a = 1$.

Answer: $y = (x+3)(x-5)$

(b) State its axis and turning point. [2 marks]

Take the midpoint of the roots.

Evaluate y at $x = 1$.

Answer: Axis $x = 1$; vertex $(1, -16)$.

3. Let $f(x) = x^2 + 6x + 2$. [4 marks]

(a) Find the minimum value by completing the square. [2 marks]

Write $f = (x+3)^2 - 7$.

Answer: Minimum -7 at $x = -3$.

(b) Solve $f(x) = 0$ exactly. [2 marks]

Isolate the square in $(x+3)^2 = 7$.

Answer: $x = -3 \pm \sqrt{7}$

4. The equation $x^2 + px + 4 = 0$ has two distinct real roots. [5 marks]

(a) Find the possible values of p. [3 marks]

Require $p^2 - 16 > 0$.

Solve the inequality on both sides of the origin.

Answer: $p < -4$ or $p > 4$

(b) For $p = -5$, find both roots. [2 marks]

Factor $x^2 - 5x + 4 = (x-1)(x-4)$.

Answer: $x = 1, 4$

5. Compare $f(x) = x^2 - 2x - 3$ and $g(x) = -2(x-1)^2 + 8$. [5 marks]

(a) Find both vertices. [2 marks]

Complete $f = (x-1)^2 - 4$.

Read g directly.

Answer: f: $(1, -4)$; g: $(1, 8)$.

(b) Find their points of intersection. [3 marks]

Set $(x-1)^2 - 4 = -2(x-1)^2 + 8$.

Then $(x-1)^2 = 4$; substitute the two x-values.

Answer: $(-1, 0)$ and $(3, 0)$.

6. For $y = 2(x+2)^2 + 3$: [4 marks]

(a) Find its y-intercept and state its range. [2 marks]

Substitute $x = 0$.

The square has minimum zero.

Answer: $(0, 11)$; $y \geq 3$.

(b) Explain why there are no real x-intercepts. [2 marks]

Solving $y = 0$ would require $(x+2)^2 = \frac{-3}{2}$, impossible for real x.

Answer: Every y-value is at least 3.

7. A straight line $y = x + 1$ meets the parabola $y = x^2 - 3x + 1$. [5 marks]

(a) Find both intersection points. [3 marks]

Set $x + 1 = x^2 - 3x + 1$.

Factor $x(x-4) = 0$, then substitute into the line.

Answer: $(0, 1)$ and $(4, 5)$.

(b) For which x is the parabola below the line? [2 marks]

Their difference is $x(x-4)$.

It is negative between its simple roots.

Answer: $0 < x < 4$

8. A quadratic passes through $(0, 3)$, $(1, 0)$ and $(3, 0)$. [5 marks]

(a) Find its rule in expanded form. [3 marks]

The roots give $a(x-1)(x-3)$.

At $x = 0$, $3 = 3a$.

Answer: $y = x^2 - 4x + 3$

(b) Find the vertex and explain its relation to the two roots. [2 marks]

The axis is $x = \frac{1+3}{2} = 2$.

Substitute $x = 2$.

Answer: $(2, -1)$; its x-coordinate is the roots' midpoint.

9. Let $p(x) = x^3 + kx^2 - 4x - 4$ and suppose $x + 1$ is a factor. [5 marks]

(a) Find k. [2 marks]

Set $p(-1) = -1 + k + 4 - 4 = 0$.

Answer: $k = 1$

(b) Factorise p completely and state all roots. [3 marks]

At $k = 1$, group $x^2(x + 1) - 4(x + 1)$.

Factor $x^2 - 4$.

Answer: $(x + 1)(x - 2)(x + 2)$; roots -2,-1,2.

10. Squares of side x cm are cut from a 12 cm by 8 cm sheet, then the sides are folded into an open box. [7 marks]

(a) Form the volume rule and its physical domain. [2 marks]

Each folded dimension must be positive.

Answer: $V = x(12 - 2x)(8 - 2x)$; $0 < x < 4$.

(b) Find every cut size giving volume 64 cubic centimetres. You may use the known root $x = 2$. [5 marks]

Set $V = 64$ and divide by 4: $x^3 - 10x^2 + 24x - 16 = 0$.

Divide by $x - 2$ to get $x^2 - 8x + 8$.

The remaining roots are $4 \pm 2\sqrt{2}$; reject the one larger than 4.

Answer: $x = 2$ or $x = 4 - 2\sqrt{2} \approx 1.172$ cm.