Quadratics and polynomial graphs - Practice 1

10 questions | 49 marks | 65 minutes

A balanced paper progressing from products and equations to graphs and polynomial models.

Equivalent correct methods earn the corresponding marks.

1. Work with quadratic products. [5 marks]

(a) Expand $(2x-3)(x+5)$. [2 marks]

Distribute: $2x^2+10x-3x-15$.

Collect like terms.

Answer: $2x^2+7x-15$

(b) Factorise $3x^2-11x+6$. [3 marks]

Split $-11x$ as $-9x-2x$.

Group $3x(x-3)-2(x-3)$.

Answer: $(3x-2)(x-3)$

2. Solve the following equations, giving every real solution. [4 marks]

(a) $x^2-5x=14$. [2 marks]

Rearrange $x^2-5x-14=(x-7)(x+2)=0$.

Answer: $x=-2,7$

(b) $2x^2=9x$. [2 marks]

Rearrange $x(2x-9)=0$.

Do not divide away the zero solution.

Answer: $x=0,\frac{9}{2}$

3. Let $f(x) = 2x^2 - 8x + 3$. [5 marks]

(a) Write f in vertex form. [3 marks]

Factor $2(x^2 - 4x) + 3$.

Complete the square inside.

Answer: $f(x) = 2(x-2)^2 - 5$

(b) State the vertex and minimum value. [2 marks]

The nonnegative square is smallest at $x = 2$.

Answer: Vertex $(2, -5)$; minimum -5.

4. Consider $x^2 + 4x - 3 = 0$. [4 marks]

(a) Find both exact roots using the quadratic formula. [3 marks]

The discriminant is 28.

$x = \frac{-4 \pm 2\sqrt{7}}{2}$.

Answer: $x = -2 \pm \sqrt{7}$

(b) Give the positive root to 3 decimal places. [1 mark]

Evaluate $-2 + \sqrt{7} = 0.645751\ldots$.

Round the final value.

Answer: 0.646

5. A family of equations is $x^2 - 4x + k = 0$. [4 marks]

(a) Find k for one repeated real root. [2 marks]

Set discriminant $16 - 4k$ equal to zero.

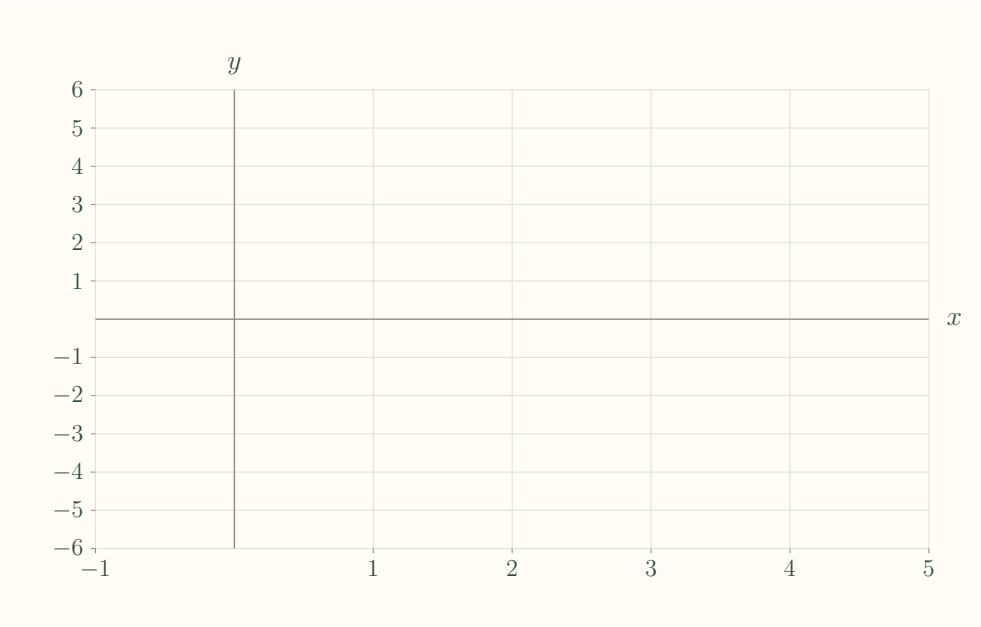
Answer: $k = 4$

(b) Find all k giving no real roots. [2 marks]

Require $16 - 4k < 0$ and solve.

Answer: $k > 4$

6. A parabola has equation $y = -(x-3)^2 + 4$. [5 marks]



(a) Find its intercepts and axis. [3 marks]

Solve $(x-3)^2 = 4$.

Substitute $x = 0$ for the y-intercept.

Answer: x-intercepts $(1, 0), (5, 0)$; y-intercept $(0, -5)$; axis $x = 3$.

(b) Sketch it and state its range. [2 marks]

The negative leading coefficient makes the vertex a maximum.

Label the features on the sketch.

Answer: $y \leq 4$; downward parabola with vertex $(3, 4)$.

7. The roots of $3x^2 - 8x + 2 = 0$ are r and s. [5 marks]

(a) Find $r + s$ and rs without solving. [2 marks]

Use $\frac{-b}{a}$ and $\frac{c}{a}$.

Answer: $r + s = \frac{8}{3}$; $rs = \frac{2}{3}$.

(b) Form a quadratic equation whose roots are $3r$ and $3s$. [3 marks]

The new sum is 8.

The new product is $9(\frac{2}{3}) = 6$.

Answer: $x^2 - 8x + 6 = 0$

8. A ball has height $h = -5t^2 + 15t + 2$ metres at t seconds after release. [6 marks]

(a) Find the maximum height and the time it occurs. [3 marks]

Rewrite $h = -5(t - 1.5)^2 + 13.25$.

Interpret the vertex.

Answer: 13.25 m at $t = 1.5$ s.

(b) Find when it reaches the ground, to 3 decimal places. [3 marks]

Set $h = 0$ and apply the quadratic formula.

Reject the negative time root.

Answer: $t = \frac{15 + \sqrt{265}}{10} \approx 3.128$ s.

9. Let $p(x) = x^3 - 4x^2 + x + 6$. [5 marks]

(a) Show that $x - 2$ is a factor and find the quadratic quotient. [3 marks]

Substitute 2: $8 - 16 + 2 + 6 = 0$.

Divide by $x - 2$.

Answer: $p(2) = 0$; quotient $x^2 - 2x - 3$.

(b) Solve $p(x) = 0$. [2 marks]

Factor the quotient as $(x - 3)(x + 1)$.

Use all three factors.

Answer: $x = -1, 2, 3$

10. Consider $g(x) = x^4 - 5x^2 + 4$. [6 marks]

(a) Solve $g(x) = 0$ over the reals. [3 marks]

Set $u = x^2$ and factor $(u - 1)(u - 4)$.

Use both roots for each u.

Answer: $x = -2, -1, 1, 2$

(b) State the y-intercept, end behaviour and whether each root crosses or touches. [3 marks]

The leading degree is even with positive coefficient.

Each linear factor has multiplicity one.

Answer: $(0, 4)$; both ends rise; all four simple roots cross.