Cubic and quartic graphs and models

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. Describe crossing/touching roots of $y = (x+1)(x-2)^2$. [2 marks]

Use multiplicities one and two.

Answer: Crosses at -1; touches at 2.

2. State end behaviour of $y = -2x^4 + 3x - 1$. [2 marks]

The leading degree is even and coefficient negative.

Answer: Both ends fall.

3. Solve $x^4 - 10x^2 + 9 = 0$. [3 marks]

Let $u = x^2$: $(u-1)(u-9) = 0$.

Use both roots of each positive u.

Answer: $x = -3,\ -1, 1, 3$

4. Solve $x^4 + x^2 - 6 = 0$ for real x. [3 marks]

The u-equation is $(u+3)(u-2)=0$.

Reject $u=-3$ for real x.

Answer: $x = \pm\sqrt{2}$

5. Cut squares of side x cm from the corners of a 12 cm by 8 cm sheet and fold a box. Give volume and domain. [3 marks]

The three dimensions must be positive.

Multiply height, length and width.

Answer: $V = x(12-2x)(8-2x)$ cubic centimetres; $0 < x < 4$.

6. A box has volume $V = x(12-2x)(8-2x)$ cubic centimetres. Find its volume when $x=1$ and when $x=2$. [2 marks]

At 1: $1(10)(6)=60$.

At 2: $2(8)(4)=64$.

Answer: 60 cubic centimetres and 64 cubic centimetres.