Functions and straight-line graphs - Practice test 5

10 questions | 50 marks | 75 minutes

A more demanding synthesis paper: justify restrictions, evaluate models and combine coordinate methods.

Equivalent correct methods earn the corresponding marks.

1. A rule is $f(x)=\frac{x^2-9}{x-3}$, with its original algebraic domain. [5 marks]

(a) Simplify the rule, retaining the input restriction. [2 marks]

Factor x^2-9 as $(x-3)(x+3)$.

Cancellation is permitted only if $x\neq 3$.

Answer: $f(x)=x+3$, with $x\neq 3$.

(b) Find $f(4)$ and explain why output 6 is missing. [3 marks]

$4+3=7$.

$x+3=6$ gives $x=3$, which is not in the original domain.

Answer: $f(4)=7$; output 6 would require the excluded input $x=3$.

2. A relation consists of the pairs $(-1,2)$, $(0,3)$ and $(1,2)$. [4 marks]

(a) Is it a function of the first coordinate? Explain. [2 marks]

Each first coordinate occurs with exactly one output.

Answer: Yes.

(b) If input and output roles are swapped, is the reversed relation a function? Explain. [2 marks]

The reversed relation includes $(2,-1)$ and $(2,1)$, so input 2 has conflicting outputs.

Answer: No.

3. A line crosses the positive axes and forms a triangle of area 12 square units with them. Its y-intercept is $(0, 6)$. [4 marks]

(a) Find its positive x-intercept. [2 marks]

Using triangle area, $(\frac{1}{2}) \times b \times 6 = 12$ gives $b = 4$.

Answer: $(4, 0)$.

(b) Find the line equation and gradient. [2 marks]

Use intercepts $(4, 0)$ and $(0, 6)$.

$\text{Gradient} = \frac{0-6}{4-0} = \frac{-3}{2}$.

Answer: $y = \frac{-3x}{2} + 6$; gradient $\frac{-3}{2}$.

4. Two lines are $y = (a+1)x - 4$ and $y = 2x + 5$. [4 marks]

(a) Find a for which they are parallel and distinct. [2 marks]

Match slopes: $a + 1 = 2$.

Their intercepts differ, so they remain distinct.

Answer: $a = 1$.

(b) Find a for which they are perpendicular. [2 marks]

The slope product must be -1: $2(a+1) = -1$.

Thus $a + 1 = \frac{-1}{2}$ and $a = \frac{-3}{2}$.

Answer: $a = \frac{-3}{2}$.

5. Two pumps together supply 27 litres per minute. Pump A supplies 3 litres per minute more than twice the rate of pump B. [5 marks]

(a) Form equations for rates a and b in litres per minute. [2 marks]

Translate the combined rate and the comparison separately.

Answer: $a + b = 27$, $a - 2b = 3$.

(b) Find both rates and check them. [3 marks]

Substitute $a = 2b + 3$: $3b + 3 = 27$ gives $b = 8$.

Then $a = 19$; $19 + 8 = 27$ and $19 = 2(8) + 3$.

Answer: A: 19 L/min; B: 8 L/min.

6. A point $P(x, y)$ is equally distant from $A(-2, 0)$ and $B(4, 0)$. [5 marks]

(a) Write an equation using squared distances and simplify it. [3 marks]

$(x+2)^2 + y^2 = (x-4)^2 + y^2$.

Expand and cancel $x^2 + y^2$: $4x + 4 = -8x + 16$, so $x = 1$.

Answer: $x = 1$.

(b) Explain the geometric meaning of this result. [2 marks]

AB is horizontal, its midpoint is $(1, 0)$, and $x = 1$ is the vertical line through that midpoint.

Answer: The locus is the perpendicular bisector of AB.

7. A simple gas model keeps pressure p times volume V equal to 200, with p in kPa and V in litres. Both quantities are positive. [5 marks]

(a) Write V as a function of p. Find V at $p = 80$ and $p = 125$. [2 marks]

Divide $pV = 200$ by positive p.

$\frac{200}{80} = 2.5$ and $\frac{200}{125} = 1.6$.

Answer: $V = \frac{200}{p}$; 2.5 L and 1.6 L.

(b) Explain why the graph is not a straight line, and state its contextual domain. [3 marks]

For example, $p = 50, 100, 150$ produces $V = 4, 2, \frac{4}{3}$, with unequal volume changes for equal pressure changes.

Only positive pressures are meaningful under the stated model.

Answer: Its changes are not constant; $p > 0$.

8. A circle has centre $(3, -2)$ and passes through $(0, 2)$. [5 marks]

(a) Find its radius and write its equation. [2 marks]

The squared distance to the given point is $(-3)^2 + 4^2 = 25$.

Answer: Radius 5; $(x-3)^2 + (y+2)^2 = 25$.

(b) Give the lower semicircle rule and its domain and range. [3 marks]

Choose the negative square-root branch.

The domain spans centre x-coordinate ±radius; the lower outputs run from -7 up to -2.

Answer: $y = -2 - \sqrt{25 - (x-3)^2}$; domain $[-2, 8]$, range $[-7, -2]$.

9. A hire company records these charges: 8 dollars at 0 hours, 18 dollars at 2 hours and 28 dollars at 4 hours. [5 marks]

(a) Find a continuous linear model fitting the observations. [2 marks]

The gradient is $\frac{18-8}{2}=5$ dollars per hour; the charge at $t=0$ is 8 dollars.

Answer: $C(t)=5t+8$.

(b) The company might charge by each started hour instead. Explain why the three observations do not settle which tariff is used. [3 marks]

The data do not show what happens at a fractional hour.

A charge measured at, for example, $t=0.5$ h would help distinguish the proposed tariffs.

Answer: Both a continuous rate and an hourly step tariff can agree at the listed whole-hour times.

10. Two villages are at $P(1,2)$ and $Q(7,10)$, on axes measured in kilometres. A radio mast must be equally distant from both. [8 marks]

(a) Find the midpoint of PQ and its gradient. [2 marks]

Average coordinates to obtain $(4,6)$.

The slope is $\frac{10-2}{7-1}=\frac{8}{6}=\frac{4}{3}$.

Answer: Midpoint $(4,6)$; gradient $\frac{4}{3}$.

(b) Find the perpendicular bisector of PQ. [3 marks]

Its gradient is $\frac{-3}{4}$ and it passes through $(4,6)$.

Use $y-6=(\frac{-3}{4})(x-4)$.

Answer: $y=\frac{-3x}{4}+9$.

(c) The mast must also lie on the y-axis. Find its position and check its distance from each village. [3 marks]

Set $x=0$ in the bisector to get $y=9$.

Squared distances are $(0-1)^2+(9-2)^2=50$ and $(0-7)^2+(9-10)^2=50$.

Answer: $(0,9)$; each distance is $\sqrt{50}=5\sqrt{2}$ km.