Functions and straight-line graphs - Practice test 4

10 questions | 44 marks | 70 minutes

A reasoning paper with parameter restrictions, a shifted reciprocal rule and geometric modelling.

Equivalent correct methods earn the corresponding marks.

1. The linear function f satisfies $f(-1)=-5$ and $f(3)=7$. [4 marks]

(a) Find the rule $f(x)$. [2 marks]

The slope is $\frac{7+5}{3+1}=3$.

Use $-5=-3+b$ to obtain $b=-2$.

Answer: $f(x)=3x-2$

(b) Compare $f(2t)$ with $2f(t)$. Are they equal for all t? [2 marks]

Substitute $2t$ into the rule for the first expression.

Double the entire output for the second; they differ by 2.

Answer: No: $f(2t)=6t-2$, while $2f(t)=6t-4$.

2. Let $h(x)=\sqrt{10-2x}$. [4 marks]

(a) Find the real domain and range without a contextual restriction. [2 marks]

Require $10-2x\geq 0$.

As x decreases without bound, the non-negative square root increases without bound.

Answer: Domain $x\leq 5$; range $y\geq 0$.

(b) Now restrict $0\leq x\leq 5$. Find the new range. [2 marks]

The rule decreases across this interval.

$h(5)=0$ and $h(0)=\sqrt{10}$; both endpoints are included.

Answer: $0\leq h(x)\leq \sqrt{10}$.

3. Consider the line $3y - 6x = 15$. [4 marks]

(a) Find its gradient and its two intercepts. [2 marks]

Divide by 3 to obtain $y = 2x + 5$.

Setting $y = 0$ gives $x = \frac{-5}{2}$.

Answer: Gradient 2; intercepts $(\frac{-5}{2}, 0)$ and $(0, 5)$.

(b) A student says $y = 2x + 1$ is perpendicular to it because the intercepts differ. Correct the statement. [2 marks]

Their gradients are both 2.

Perpendicularity depends on the gradients, not whether intercepts differ.

Answer: They are distinct parallel lines, not perpendicular.

4. For a real number p, two points are $A(p, 1)$ and $B(p, 6)$. [4 marks]

(a) State the equation and gradient of AB. [2 marks]

Both points have the same x-coordinate and distinct y-coordinates.

Answer: $x = p$; its gradient is undefined.

(b) Find the line perpendicular to AB through $(0, -3)$. [2 marks]

The perpendicular to a vertical line is horizontal.

Answer: $y = -3$.

5. A laboratory mixes 10% and 30% saline solutions to make 20 L of a 22% solution. Assume volumes add. [5 marks]

(a) Form equations for the litres x and y of the two solutions. [2 marks]

The final salt-equivalent volume is $0.22 \times 20 = 4.4$ L.

Answer: $x + y = 20$, $0.10x + 0.30y = 4.4$.

(b) Solve and check the mixture. [3 marks]

Use $x = 20 - y$: $2 - 0.1y + 0.3y = 4.4$ gives $y = 12$.

Then $x = 8$; $0.8 + 3.6 = 4.4$ L of salt-equivalent content.

Answer: 8 L of 10% solution and 12 L of 30% solution.

6. Three points are $A(-4,1)$, $B(2,9)$, $C(2,1)$. [4 marks]

(a) Calculate all three side lengths. [2 marks]

AC is horizontal and BC is vertical.

$AB = \sqrt{6^2 + 8^2} = 10$.

Answer: $AC = 6$, $BC = 8$, $AB = 10$.

(b) Explain why the triangle is right-angled and find the midpoint of AB. [2 marks]

AC and BC are horizontal and vertical, so they are perpendicular.

The midpoint averages the endpoints of AB.

Answer: The right angle is at C; midpoint $(-1,5)$.

7. A shifted reciprocal rule is $y = \frac{6}{x-2} + 1$. [5 marks]

(a) State the asymptotes, domain and range. [2 marks]

The denominator cannot be zero.

The reciprocal term cannot be zero but approaches zero for large magnitude x.

Answer: Asymptotes $x = 2$ and $y = 1$; domain $x \neq 2$; range $y \neq 1$.

(b) Find both axis intercepts. [3 marks]

At $y = 0$: $\frac{6}{x-2} = -1$ gives $x = -4$.

At $x = 0$: $y = -3 + 1 = -2$.

Answer: $(-4, 0)$ and $(0, -2)$.

8. A circle has centre $(-2,1)$ and passes through $(1,3)$. [5 marks]

(a) Write its equation. [2 marks]

The squared radius is $3^2 + 2^2 = 13$.

Answer: $(x+2)^2 + (y-1)^2 = 13$.

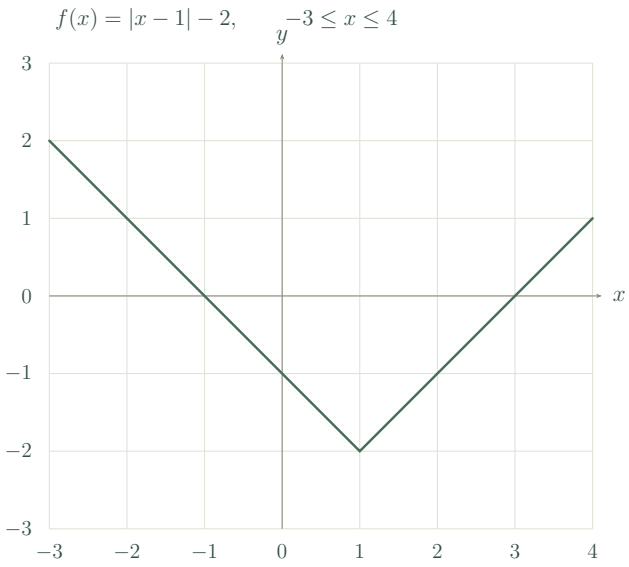
(b) Find its exact x-intercepts. [3 marks]

Set $y = 0$: $(x+2)^2 = 12$.

Take both square roots: $x + 2 = \pm 2\sqrt{3}$.

Answer: $(-2-2\sqrt{3}, 0)$ and $(-2+2\sqrt{3}, 0)$.

9. A V-shaped function is $f(x) = |x - 1| - 2$ on $-3 \leq x \leq 4$. Here $|u|$ means the non-negative distance of u from zero. [4 marks]



(a) Find its minimum point and its range. [2 marks]

The absolute value is smallest at $x = 1$.

Endpoint outputs are 2 and 1; hence the largest value on this domain is 2.

Answer: Minimum $(1, -2)$; range $[-2, 2]$.

(b) Does its repeated output at $x = 0$ and $x = 2$ prevent it being a function? Explain. [2 marks]

The function definition does not require different inputs to have different outputs.

Answer: No; both outputs are -1, but each input still has one output.

10. A linear taxi tariff charges 17.50 dollars for 5 km and 27.50 dollars for 9 km. [5 marks]

(a) Find a fare rule $C(d)$, explaining both coefficients. [2 marks]

The slope is $\frac{27.50-17.50}{9-5}=2.50$.

Using $d=5$ gives $17.50=12.50+b$, so $b=5$.

Answer: $C=2.5d+5$; rate 2.50 dollars/km and initial fee 5 dollars.

(b) What is the greatest distance costing at most 25 dollars under this tariff? [3 marks]

$2.5d+5\le 25$ gives $d\le 8$.

Distance is non-negative, so the permitted distances are $0\le d\le 8$.

Answer: 8 km.