Functions and straight-line graphs - Practice test 3

10 questions | 43 marks | 70 minutes

Use functions to connect algebra with measurement and practical restrictions. Justify choices of models.

Equivalent correct methods earn the corresponding marks.

1. The rule is $f(x)=\frac{3}{2-x}$. [4 marks]

(a) Evaluate $f(-1)$ and $f(5)$. [2 marks]

$f(-1)=\frac{3}{3}=1$; $f(5)=\frac{3}{-3}=-1$.

Answer: 1 and -1.

(b) Solve $f(x)=6$ and state the excluded input of the original rule. [2 marks]

$3=6(2-x)$, so $6x=9$ and $x=\frac{3}{2}$.

The original denominator is zero at $x=2$.

Answer: $x=\frac{3}{2}$; $x=2$ is excluded.

2. A rectangle has fixed width 4 cm and variable positive length l cm. [5 marks]

(a) Write area A as a function of l. Name the independent and dependent variables and their units. [2 marks]

Multiply length by width; area responds to the chosen length.

Answer: $A=4l$; input l in cm, output A in cm².

(b) State suitable domains and ranges for $A(l)$ and perimeter $P(l)$. [3 marks]

A non-degenerate rectangle has positive length.

As l approaches zero, the perimeter approaches 8 without reaching it.

Answer: $l>0$, $A>0$; $P=2l+8$ has $P>8$.

3. A straight line has equation $2x - 3y = 9$. [4 marks]

(a) Find its gradient and intercepts. [2 marks]

Rearrange as $y = \frac{2x}{3} - 3$.

Set $y = 0$ for the x-intercept.

Answer: Gradient $\frac{2}{3}$; intercepts $(\frac{9}{2}, 0)$, $(0, -3)$.

(b) Find the point on this line with $y = 5$. [2 marks]

$2x - 15 = 9$ gives $x = 12$.

Answer: $(12, 5)$.

4. Line L passes through $(1, 2)$ and $(5, 4)$. [4 marks]

(a) Find its equation. [2 marks]

$\text{Gradient} = \frac{4-2}{5-1} = \frac{1}{2}$.

Use $2 = \frac{1}{2} + c$, giving $c = \frac{3}{2}$.

Answer: $y = \frac{x}{2} + \frac{3}{2}$

(b) Find the line perpendicular to L through $(3, 1)$. [2 marks]

The perpendicular slope is -2.

$y - 1 = -2(x - 3)$.

Answer: $y = -2x + 7$

5. A car park charges 8 dollars for a van and 5 dollars for a car. There are 13 vehicles and the total charge is 80 dollars. [5 marks]

(a) Form simultaneous equations for the vehicle counts. [2 marks]

v and c count vans and cars.

Answer: $v + c = 13,\ 8v + 5c = 80.$

(b) Solve and interpret the solution. [3 marks]

Subtract five times the count equation from the charge equation: $3v = 15.$

Then $v = 5,\ c = 8$; their charges sum to $40 + 40 = 80$ dollars.

Answer: 5 vans and 8 cars.

6. A quadrilateral has $A(0, 1),\ B(4, 1),\ C(4, 4)$ and $D(0, 4)$. [4 marks]

(a) Calculate the lengths of diagonals AC and BD. [2 marks]

AC has changes 4 and 3, so length 5.

BD has changes -4 and 3, so also length 5.

Answer: Both are 5 units.

(b) Calculate both diagonal midpoints. State what the matching midpoints show. [2 marks]

Average corresponding endpoint coordinates for each diagonal.

A common midpoint divides each segment into equal halves.

Answer: Both midpoints are $(2, \frac{5}{2})$, so the diagonals bisect each other.

7. A job needs 60 worker-hours. Assume all workers have the same constant work rate and can work together without interference. [4 marks]

(a) Write the completion time t in hours as a function of the positive number of workers w. Find t for 5 workers. [2 marks]

The product $wt = 60$ under the stated model.

At $w = 5$, $t = 12$.

Answer: $t = \frac{60}{w}$; 12 hours.

(b) Find t for 8 workers and explain one limitation of the model. [2 marks]

$\frac{60}{8} = 7.5$.

The calculation assumes extra workers do not reduce anyone's rate.

Answer: 7.5 hours; coordination or limited workspace may prevent perfect inverse scaling.

8. A circle is centred at $(2, -1)$ and passes through $(5, 3)$. [4 marks]

(a) Find its radius and equation. [2 marks]

The radius is $\sqrt{3^2 + 4^2} = 5$.

Insert centre and radius into the standard form.

Answer: Radius 5; $(x-2)^2 + (y+1)^2 = 25$.

(b) Find the leftmost, rightmost, lowest and highest points. [2 marks]

Move one radius horizontally or vertically from the centre.

Answer: $(-3, -1)$, $(7, -1)$, $(2, -6)$, $(2, 4)$.

9. The table describes part of a reciprocal function. [4 marks]

x	-2	-1	0	1	2
y	-4	-8	?	8	4

(a) Find its rule and complete the missing entry if possible. [2 marks]

Every listed non-zero pair has product 8.

Division by zero is undefined; do not insert 0 as the output.

Answer: $y = \frac{8}{x}$; no output is defined at $x = 0$.

(b) State whether this is a function of x and describe its two branches. [2 marks]

Each non-zero input gives one output.

Because the product is positive, input and output have matching signs.

Answer: Yes on $x \neq 0$; the branches lie in quadrants I and III.

10. An engineer proposes a linear model $y = ax + b$. It must pass through measurements $(2, 11)$ and $(6, 23)$. [5 marks]

(a) Determine a and b. [2 marks]

The gradient is $\frac{23 - 11}{6 - 2} = 3$.

Use $11 = 3(2) + b$ to get $b = 5$.

Answer: $a = 3$, $b = 5$.

(b) A third measurement is $(8, 32)$. Does the exact model pass through it? Give a careful conclusion. [3 marks]

$3(8) + 5 = 29 \neq 32$.

The exact linear rule fails to fit all three measurements. Measurement uncertainty or a different model would need investigation; two points alone do not prove a real process is linear.

Answer: No; it predicts $y = 29$, three units lower.