Functions and straight-line graphs - Practice test 2

10 questions | 42 marks | 70 minutes

Use symbolic rules, intercepts and contextual restrictions. Explanations matter as well as calculations.

Equivalent correct methods earn the corresponding marks.

1. Let $f(x) = x^2 - 2x + 3$. [4 marks]

(a) Find $f(-1)$. [2 marks]

$(-1)^2 - 2(-1) + 3 = 6.$

Answer: 6

(b) Simplify $f(t+1)$. [2 marks]

$(t+1)^2 - 2(t+1) + 3 = t^2 + 2t + 1 - 2t - 2 + 3 = t^2 + 2.$

Answer: $t^2 + 2$

2. Consider the reciprocal rule $g(x) = \frac{4}{x-3}$ and the relation $R = \{(2, 1), (2, 5), (4, 3)\}$. [4 marks]

(a) State the domain and range of g. [2 marks]

The denominator must be non-zero.

A non-zero numerator cannot produce zero.

Answer: All real x except 3; all real outputs except 0.

(b) Explain whether R is a function of x. [2 marks]

Input 2 is paired with outputs 1 and 5.

Answer: It is not.

3. A line passes through $(-2, -1)$ and $(4, 8)$. [4 marks]

(a) Find its equation. [2 marks]

$\text{Gradient} = \frac{8+1}{4+2} = \frac{3}{2}$.

Using $(-2, -1)$, $-1 = -3 + c$ gives $c = 2$.

Answer: $y = \frac{3x}{2} + 2$

(b) Find both axis intercepts. [2 marks]

Set $y = 0$ to get $x = \frac{-4}{3}$; set $x = 0$ to get $y = 2$.

Answer: $(\frac{-4}{3}, 0)$ and $(0, 2)$.

4. Use gradients to construct the required lines. [4 marks]

(a) Find the line perpendicular to $3x + 2y = 8$ through $(3, -2)$. [2 marks]

The original gradient is $\frac{-3}{2}$, so the perpendicular gradient is $\frac{2}{3}$.

Use $y + 2 = (\frac{2}{3})(x - 3)$.

Answer: $y = \frac{2x}{3} - 4$

(b) Give the lines parallel and perpendicular to $x = -5$ through $(2, 3)$. [2 marks]

A line parallel to a vertical line is vertical; its perpendicular is horizontal.

Answer: Parallel: $x = 2$. Perpendicular: $y = 3$.

5. A collection contains 20 coins, each either a 2-dollar coin or a 50-cent coin. Their total value is 22 dollars. [5 marks]

(a) Form equations for the two numbers of coins. [2 marks]

Let a count 2-dollar coins and b count 50-cent coins.

Answer: $a + b = 20$, $2a + 0.5b = 22$.

(b) Solve and check the numbers. [3 marks]

Substitute $b = 20 - a$: $2a + 10 - 0.5a = 22$, giving $a = 8$.

Then $b = 12$; the value is $16 + 6 = 22$ dollars.

Answer: 8 two-dollar coins and 12 fifty-cent coins.

6. A segment has endpoint $A(-3, 4)$ and midpoint $M(2, -1)$. [4 marks]

(a) Find its other endpoint B. [2 marks]

$\frac{-3+x}{2} = 2$ gives $x = 7$.

$\frac{4+y}{2} = -1$ gives $y = -6$.

Answer: $(7, -6)$.

(b) Find the exact length AB. [2 marks]

The coordinate changes are 10 and -10.

$AB = \sqrt{100 + 100} = 10\sqrt{2}$.

Answer: $10\sqrt{2}$ units.

7. y is inversely proportional to x. When $x = 4$, $y = -5$. [4 marks]

(a) Find the rule and evaluate y at $x = -2$. [2 marks]

The constant product is $4(-5) = -20$.

At $x = -2$, $y = \frac{-20}{-2} = 10$.

Answer: $y = \frac{-20}{x}$; $y = 10$.

(b) State the asymptotes and explain whether y can be zero. [2 marks]

No finite permitted input makes $\frac{-20}{x}$ equal zero.

Answer: Asymptotes $x = 0$ and $y = 0$; y is never zero.

8. The circle relation is $x^2 + (y+1)^2 = 16$. [5 marks]

(a) State its centre and radius. [2 marks]

Compare with standard circle form.

Answer: $(0, -1)$, radius 4.

(b) Write its upper semicircle as a function and state its domain and range. [3 marks]

Choose the positive square-root branch.

The highest output is $-1 + 4 = 3$, and endpoint outputs are -1.

Answer: $y = -1 + \sqrt{16 - x^2}$; domain $[-4, 4]$, range $[-1, 3]$.

9. A delivery service proposes a linear fee model using the supplied table. [4 marks]

Distance d (km)	2	5	8
Fee C (dollars)	9	15	21

(a) Find the fee rule $C(d)$. [2 marks]

A 3 km increase adds 6 dollars, so the rate is 2 dollars/km.

At $d=2$, $9=4+c$ gives $c=5$.

Answer: $C=2d+5$ dollars.

(b) Predict the fee for 10 km. State one assumption behind your prediction. [2 marks]

$C(10)=2(10)+5=25$.

The observed data alone do not prove that the tariff continues unchanged.

Answer: 25 dollars; the same linear tariff must apply at 10 km.

10. Two lines are $y=(k-1)x+2$ and $y=3x-5$. [4 marks]

(a) Find k for which they are parallel. Can they ever be the same line? [2 marks]

Equal slopes require $k-1=3$.

Their fixed y-intercepts 2 and -5 differ.

Answer: $k=4$; they cannot coincide.

(b) Find their intersection when $k=2$. [2 marks]

Then the first line is $y=x+2$.

$x+2=3x-5$ gives $x=\frac{7}{2}$, and $y=\frac{11}{2}$.

Answer: $(\frac{7}{2}, \frac{11}{2})$.