Functions and straight-line graphs - Practice test 1

10 questions | 46 marks | 70 minutes

A balanced paper on rules, graphs, linear geometry and practical models. Give exact values unless a question asks otherwise.

Equivalent correct methods earn the corresponding marks.

1. Let $f(x) = 2x - 3$. [6 marks]

(a) Find $f(-2)$. [2 marks]

$f(-2) = 2(-2) - 3 = -7$.

Answer: -7

(b) Simplify $f(a + 1)$. [2 marks]

$2(a + 1) - 3 = 2a - 1$.

Answer: $2a - 1$

(c) Find the input when the output is 11. [2 marks]

Solve $2x - 3 = 11$, giving $x = 7$.

Answer: $x = 7$

2. Consider functions and their possible inputs. [4 marks]

(a) State the real domain and range of $g(x) = \sqrt{x + 2}$. [2 marks]

Require $x + 2 \geq 0$. The square root gives every non-negative output.

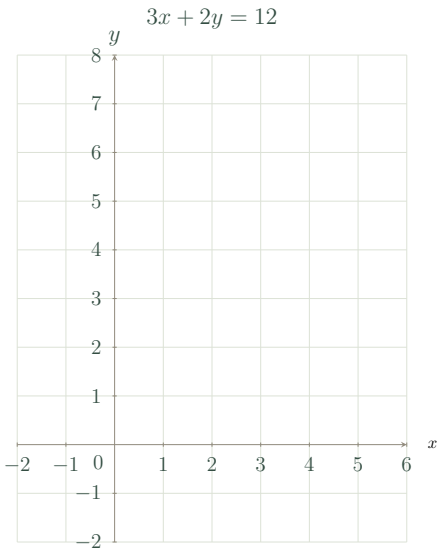
Answer: Domain $x \geq -2$; range $y \geq 0$.

(b) Does $\{(-1, 3), (0, 3), (2, 5)\}$ define a function of its first coordinate? Explain. [2 marks]

Each input has one output; repeated output 3 is allowed.

Answer: Yes.

3. A line has equation $3x + 2y = 12$. [4 marks]



(a) Find its gradient and y-intercept. [2 marks]

Rearrange to $y = \frac{-3x}{2} + 6$.

Answer: Gradient $\frac{-3}{2}$; y-intercept $(0, 6)$.

(b) Find its x-intercept. Sketch the line on the grid, labelling both intercepts. [2 marks]

Set $y = 0$, so $3x = 12$ and $x = 4$.

Plot the two intercepts and join with a straight line.

Answer: x-intercept $(4, 0)$; draw the line through $(4, 0)$ and $(0, 6)$.

4. The line L passes through $A(-1, 4)$ and $B(3, -4)$. [6 marks]

(a) Find an equation of L. [2 marks]

$\text{Gradient} = \frac{-4-4}{3+1} = -2$.

Using A gives $4 = 2 + c$, so $c = 2$.

Answer: $y = -2x + 2$

(b) Find the line parallel to L through $(2, 5)$. [2 marks]

Keep gradient -2: $y - 5 = -2(x - 2)$.

Answer: $y = -2x + 9$

(c) Find the line perpendicular to L through $(2, 5)$. [2 marks]

The perpendicular gradient is $\frac{1}{2}$.

$y - 5 = \frac{x-2}{2}$.

Answer: $y = \frac{x}{2} + 4$

5. A performance sells 9 tickets. Adult tickets cost 12 dollars and student tickets cost 7 dollars. Total revenue is 83 dollars. [5 marks]

(a) Define variables and form two equations. [2 marks]

Let a and s count adult and student tickets respectively.

Answer: $a + s = 9$ and $12a + 7s = 83$.

(b) Find the number of each type and check your answer. [3 marks]

Use $s = 9 - a$: $12a + 63 - 7a = 83$, so $a = 4$.

Then $s = 5$; $48 + 35 = 83$ and $4 + 5 = 9$.

Answer: 4 adult and 5 student tickets.

6. A segment joins $A(-2, 1)$ to $B(4, 9)$. [4 marks]

(a) Find its exact length. [2 marks]

The changes are 6 and 8, so $\text{length} = \sqrt{36 + 64} = 10$.

Answer: 10 units.

(b) Find its midpoint. [2 marks]

Average corresponding coordinates: $(\frac{-2+4}{2}, \frac{1+9}{2})$.

Answer: $(1, 5)$.

7. A rectangle has fixed area 36 square metres. Its positive length is l metres and its width is w metres. [4 marks]

(a) Write w as a function of l and find w when $l = 9$. [2 marks]

$lw = 36$, so $w = \frac{36}{l}$; substitute $l = 9$.

Answer: $w = \frac{36}{l}$ for $l > 0$; $w = 4$ m.

(b) Find l when $w = 3$. Explain what happens to w when l doubles. [2 marks]

$l = \frac{36}{3} = 12$.

Replacing l by $2l$ gives $\frac{36}{2l}$, half the original width.

Answer: $l = 12$ m; w halves.

8. A circle has centre $(-1, 2)$ and radius 3. [4 marks]

(a) Write its equation. [2 marks]

Use the centre-radius form with $r^2 = 9$.

Answer: $(x + 1)^2 + (y - 2)^2 = 9$.

(b) State the full circle range and decide whether $(2, 2)$ is on the circle. [2 marks]

The extreme y-values are 2 ± 3.

At $(2, 2)$, the left side is $3^2 + 0 = 9$.

Answer: Range $-1 \le y \le 5$; the point is on it.

9. A tank drains at a constant rate. The table shows its remaining volume. [4 marks]

Remaining water

Time t (min)	0	5	10
Volume V (L)	120	90	60

(a) Find a linear model $V(t)$, with units. [2 marks]

The change is -30 L in 5 min, giving -6 L/min.

At $t=0$ the volume is 120 L.

Answer: $V=120-6t$, V in litres and t in minutes.

(b) Find when the tank becomes empty and state a sensible domain. [2 marks]

Solve $120-6t=0$.

The physical draining model stops once the volume reaches zero.

Answer: 20 min; $0\le t\le 20$.

10. Hire company A charges $A(h)=8+4h$ dollars and company B charges $B(h)=20+2h$ dollars for non-negative hire time h in hours. [5 marks]

(a) Find when the charges are equal. [2 marks]

$8+4h=20+2h$ gives $h=6$.

Substitute into either rule.

Answer: 6 hours; both cost 32 dollars.

(b) Explain which company is cheaper before and after that time. [3 marks]

A has the smaller starting charge but larger hourly rate.

The difference $A-B=2h-12$ changes sign at $h=6$.

Answer: A for $0\le h<6$; B for $h>6$.