Networks, scheduling and assignment - Test 6

10 questions | 60 marks | 75 minutes

A more demanding applications paper combining network sensitivity, resource assumptions and assignment-model choices.

Equivalent correct methods earn the corresponding marks.

1. Use the school-garden installation project. [6 marks]

Installing a school garden; durations are in days.

Activity	Duration	Immediate predecessors
A	3	None
B	5	None
C	4	A
D	2	A
E	3	B and C
F	6	D
G	2	E and F

(a) Complete the earliest activity schedule. [4 marks]

E waits for B and C; F follows D.

G waits for E and F.

Answer: A: 0-3, B: 0-5, C: 3-7, D: 3-5, E: 7-10, F: 5-11, G: 11-13.

(b) Find the critical path and minimum completion. [2 marks]

$3+2+6+2=13$; $\mathrm{A}-\mathrm{C}-\mathrm{E}-\mathrm{G}=12$.

Answer: A-D-F-G;13 days.

2. Use the garden project with completion at day 13. [7 marks]

Installing a school garden; durations are in days.

Activity	Duration	Immediate predecessors
A	3	None
B	5	None
C	4	A
D	2	A
E	3	B and C
F	6	D
G	2	E and F

(a) Calculate the latest start and float for B, C, E. [4 marks]

G latest 11; F latest 5; E latest 8.

C latest $8-4=4$; B latest $8-5=3$.

Answer: B: $\text{LS}=3,\ \text{float}=3$; C: $\text{LS}=4,\ \text{float}=1$; E: $\text{LS}=8,\ \text{float}=1$ day.

(b) C and E are each extended by 1 day. Find the new completion and explain why adding their original floats is misleading. [3 marks]

A-C-E-G becomes $3+5+4+2=14$.

The original one-day allowance lies on their shared branch and cannot be spent twice.

Answer: 14 days.

3. In the garden project, replace B's duration by $x \geq 0$ days. [5 marks]

Installing a school garden; durations are in days.

Activity	Duration	Immediate predecessors
A	3	None
B	5	None
C	4	A
D	2	A
E	3	B and C
F	6	D
G	2	E and F

(a) Show that the project completion is $T = \max(13, x + 5)$. [3 marks]

Identify the two competing final-branch finishes before adding G.

Answer: E finishes $\max(x, 7) + 3$; F finishes 11; G adds 2, giving $\max(x + 5, 12, 13) = \max(13, x + 5)$.

(b) Find the largest x that preserves the original finish; then find T when $x = 9$. [2 marks]

Require $x + 5 \leq 13$.

Then $\max(13, 14) = 14$.

Answer: Largest $x = 8$; at $x = 9$, $T = 14$.

4. Possible garden time reductions are assessed using the original table. [6 marks]

Installing a school garden; durations are in days.

Activity	Duration	Immediate predecessors
A	3	None
B	5	None
C	4	A
D	2	A
E	3	B and C
F	6	D
G	2	E and F

(a) Reduce F from 6 to 4 days. Find the new completion and critical route. [3 marks]

F then finishes 9, but E finishes 10; G ends 12.

Answer: 12 days; A-C-E-G.

(b) Make that reduction and also reduce C from 4 to 3 days. Find the finish and all critical routes. [3 marks]

F finishes $3+2+4=9$.

E finishes $\max(5,3+3)+3=9$; G adds 2.

Answer: 11 days; A-D-F-G and A-C-E-G.

5. For this garden project, A and B need the same single excavator. Other activities have independent resources. [6 marks]

Installing a school garden; durations are in days.

Activity	Duration	Immediate predecessors
A	3	None
B	5	None
C	4	A
D	2	A
E	3	B and C
F	6	D
G	2	E and F

(a) Schedule A before B and find the earliest completion. [3 marks]

A: 0-3, B: 3-8, C: 3-7, D: 3-5, E: 8-11, F: 5-11, G: 11-13.

Answer: 13 days.

(b) Schedule B before A and compare. [3 marks]

B: 0-5, A: 5-8, C: 8-12, D: 8-10, E: 12-15, F: 10-16, G: 16-18.

Answer: 18 days; A-first is better.

6. The four-worker assignment costs are shown. A training payment of $1 is added only to the selected job of worker B, regardless of which job B receives. [5 marks]

Assignment costs ($). Each worker does one job.

Worker	1	2	3	4
A	8	3	7	6
B	5	9	4	8
C	6	7	2	5
D	7	4	8	3

(a) Find the new optimum assignment and total. [3 marks]

Every complete assignment uses B once and gains $1.

The old optimum $13 becomes $14 without changing its ranking.

Answer: A→2, B→1, C→3, D→4;$14.

(b) A colleague instead adds $1 to all four selected entries. Explain the numerical difference. [2 marks]

A row-specific change affects one chosen entry; a whole-table change affects four.

Answer: That would add $4 to each total, giving $17 at the optimum.

7. Two workers must each take one different job. The time table is A:$[1, 6]$ and B:$[6, 10]$, in hours. Work starts simultaneously. [5 marks]

(a) Find the assignment minimising total worker-hours. [2 marks]

The alternative total is $6 + 6 = 12$.

Answer: A→1, B→2; total 11 hours.

(b) Find the assignment minimising the time when both jobs are finished. [2 marks]

The first allocation's finish is $\max(1, 10) = 10$; the alternative's is $\max(6, 6) = 6$.

Answer: A→2, B→1; finish after 6 hours.

(c) Explain which assignment the ordinary sum-cost Hungarian model chooses. [1 mark]

The algorithm optimises the objective encoded in the model.

Answer: It chooses the 11-hour total, not necessarily the earliest joint finish.

8. The team profits are in $1,000 units. TeamB cannot take contract 1. [6 marks]

Contract profit in $1,000 units.

Team	1	2	3	4
A	14	18	12	11
B	16	13	15	12
C	10	16	14	17
D	15	11	18	14

(a) Give the best remaining allocation and total profit. [4 marks]

$18 + 15 + 17 + 15 = 65$; enumeration of permitted assignments confirms the maximum.

Answer: A→2, B→3, C→4, D→1;$65,000.

(b) An outside specialist could take contract 1 for a net profit of $15,500, with the other three teams free to cover three remaining contracts. Explain why this is not automatically comparable with the four-team table. [2 marks]

The original optimum cannot be adjusted by replacing one number without defining the new feasible assignments.

Answer: It changes the available resources and one-to-one model; the eligible remaining teams, idle-team rule and other profits must be specified.

9. For the two-worker/three-job cost table below, omission penalties are permitted. [8 marks]

Costs ($), including a permitted unfilled-job penalty row.

Resource	1	2	3
A	6	4	8
B	5	7	3
Unfilled	10	2	7

(a) Find the minimum total including penalties. [3 marks]

Original selected entries $6+3+2=11$.

Answer: $11: A→1, B→3, job 2 unfilled.

(b) The client now makes job 2 compulsory but permits either job 1 or job 3 to remain unfilled at its original penalty. Find the new optimum. [4 marks]

Exclude every assignment with the dummy in column 2.

Remaining totals 20,16,17,25; minimum 16.

Answer: $16: A→2, B→1, job 3 unfilled.

(c) State what changed in the recommendation. [1 mark]

The constraint change removes the old optimum.

Answer: Job 2 is now done by A; job 3 is left unfilled, and the modelled total rises by $5.

10. A final report states: 'Our project will certainly finish in 13 days, and the assignment algorithm guarantees the cheapest real-world outcome.' [6 marks]

(a) Rewrite the project conclusion accurately. [2 marks]

A deterministic model is not a guarantee about uncertain conditions.

Answer: The dependency-based minimum is 13 days under the supplied durations and resource assumptions; uncertainties or additional restrictions may increase it.

(b) Rewrite the assignment conclusion accurately. [2 marks]

It does not account for missing costs, resource conflicts or uncertain inputs unless they are modelled.

Answer: The chosen allocation minimises the stated table objective among feasible pairings under the stated assumptions.

(c) Give one targeted check for each model. [2 marks]

The checks should address an actual modelling risk, not merely repeat the same calculation.

Answer: For example rescan with a plausible duration increase; and re-solve after updating a disputed or unavailable pairing.