Networks, scheduling and assignment - Test 4

10 questions | 54 marks | 75 minutes

Use inverse scheduling and expediting decisions, then justify assignment reductions and interpret operational restrictions.

Equivalent correct methods earn the corresponding marks.

1. Use the hall-restoration project. [7 marks]

Restoring a hall; durations are in days.

Activity	Duration	Immediate predecessors
A	4	None
B	6	None
C	3	A
D	5	A and B
E	4	C
F	2	D and E

(a) Find the earliest starts and finishes. [4 marks]

D waits for both A and B.

F waits for both D and E.

Answer: A: 0-4, B: 0-6, C: 4-7, D: 6-11, E: 7-11, F: 11-13.

(b) List every critical path. [3 marks]

Each takes 13 days.

A-D-F takes 11.

Answer: A-C-E-F and B-D-F.

2. In the hall project, replace D's duration 5 by a nonnegative variable x. [7 marks]

Restoring a hall; durations are in days.

Activity	Duration	Immediate predecessors
A	4	None
B	6	None
C	3	A
D	5	A and B
E	4	C
F	2	D and E

(a) Write the project finish in terms of x. [3 marks]

The C/E route remains $4+3+4+2=13$.

D starts at 6, so its route including F ends $6+x+2=x+8$.

Answer: $T=\max(13, x+8)$ days.

(b) Find the greatest x that still permits a day-14 finish. [2 marks]

Require $x+8\le 14$, giving $x\le 6$.

Answer: 6 days.

(c) For which x is the original 13-day minimum unchanged? [2 marks]

Require $x+8\le 13$.

Answer: $0\le x\le 5$.

3. The hall project may be expedited using only these options: reduce B by 1 day for $90; reduce C by 1 day for $100; reduce F by 1 day for $250. Each option is available at most once. [6 marks]

Restoring a hall; durations are in days.

Activity	Duration	Immediate predecessors
A	4	None
B	6	None
C	3	A
D	5	A and B
E	4	C
F	2	D and E

(a) Explain why reducing B alone does not shorten the project. [2 marks]

Only the B/D route is shortened.

Answer: A-C-E-F remains 13 days.

(b) Find the least extra cost to achieve completion in 12 days. [3 marks]

Both separate critical branches then finish their work by 10, and F ends 12.

Reducing shared F also works but costs 250.

Answer: $190: reduce both B and C.

(c) State an assumption behind this recommendation. [1 mark]

The new durations and resource conditions must be feasible.

Answer: The stated reductions are achievable without adding other delays or dependencies.

4. A project has paths P-R-T of 16 days and Q-S-T of 16 days. T is the only shared activity; other routes are shorter. [4 marks]

(a) Would reducing only R by 3 days reduce the finish? Explain. [2 marks]

A second critical path remains.

Answer: No; Q-S-T still takes 16.

(b) Reduce T by 2 days. Find the new finish, assuming shorter routes remain below 14. [2 marks]

The shared activity lies on both controlling paths.

Answer: 14 days.

5. Use the four-worker assignment costs. [6 marks]

Assignment costs ($). Each worker does one job.

Worker	1	2	3	4
A	8	3	7	6
B	5	9	4	8
C	6	7	2	5
D	7	4	8	3

(a) State an optimum and original cost. [3 marks]

$3+5+2+3=13$; enumeration confirms optimum.

Answer: A→2, B→1, C→3, D→4;$13.

(b) Add $2 to every entry of row A and subtract $1 from every entry of column 4. Does the optimal pairing change? Give its new total. [3 marks]

A complete assignment uses one entry from row A and one from column 4.

Answer: No; every total rises by $2-1=1$, so the minimum becomes $14.

6. A reduced 3×3 matrix has rows $\begin{bmatrix} 2 & 0 & 2 \\ 1 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix}$. [6 marks]

(a) Give a minimum two-line cover of its zeros and the smallest uncovered entry. [2 marks]

These lines cover every zero; one line cannot cover both row 3/column 1 and row 1/column 2.

Answer: Cover column 2 and row 3; smallest uncovered entry 1.

(b) Carry out the adjustment and select independent zeros. [3 marks]

Subtract 1 from uncovered entries; add 1 at row 3/column 2.

Answer: $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 4 \\ 0 & 1 & 0 \end{bmatrix}$; choose row 1→2, row 2→1, row 3→3.

(c) Explain why the adjusted zeros do not tell you the actual original cost. [1 mark]

Reductions shift every complete total.

Answer: They identify pairings; original costs must be read from the untransformed table.

7. Maximise the contract profits below, but team B cannot take contract 1. [6 marks]

Contract profit in \$1,000 units.

Team	1	2	3	4
A	14	18	12	11
B	16	13	15	12
C	10	16	14	17
D	15	11	18	14

(a) Find a feasible maximum-profit assignment and profit. [4 marks]

Original total $18 + 15 + 17 + 15 = 65$.

Enumeration excluding B→1 confirms this is largest.

Answer: A→2, B→3, C→4, D→1; $65 \text{ units} = \$65{,}000$.

(b) Compare it with the unrestricted optimum. [2 marks]

The unrestricted assignment uses B→1 and is no longer feasible.

Answer: Unrestricted 69 units; the restriction costs $4 \text{ units} = \$4{,}000$ predicted profit.

8. Two workers have costs A:$[6, 4, 8]$ and B:$[5, 7, 3]$. All three jobs are compulsory, and each person can do at most one. [4 marks]

(a) Explain why the original assignment problem is infeasible. [2 marks]

A dummy worker does not provide real capacity.

Answer: Two people cannot cover three distinct compulsory jobs under the one-job limit.

(b) Give two operational changes that could make it feasible. [2 marks]

Either changes the original constraints and requires a new model.

Answer: For example add a third worker, or permit a worker to do more than one job in a revised schedule.

9. A square cost problem contains negative entries because some pairings earn a rebate. The smallest entry is -4. [4 marks]

(a) Explain why adding 4 to every entry preserves the optimum. [2 marks]

The same shift applies to every feasible complete assignment.

Answer: Every complete n-row total increases by 4n.

(b) A 3-row shifted problem has minimum total 20. Find the original minimum. [2 marks]

Subtract $3 \times 4 = 12$ from 20.

Answer: 8.

10. A project model assumes a task takes exactly 6 days, but its plausible duration is 5-9 days. [4 marks]

(a) Describe a defensible scheduling check. [2 marks]

A duration range should not be replaced by an unexplained guarantee.

Answer: Recalculate the network at the plausible durations, especially 9, and identify changes in completion and critical routes.

(b) The task is not on the current critical path. Is checking it unnecessary? Explain. [2 marks]

Current non-critical status applies only to the current inputs.

Answer: No; an increase can exceed its float and create a new critical path.