Networks, scheduling and assignment - Test 3

10 questions | 58 marks | 75 minutes

Analyse multiple critical paths, constrained resources and assignments with unavailable or unfilled tasks.

Equivalent correct methods earn the corresponding marks.

1. Use the science-display project. [7 marks]

Preparing a science display; all durations are in days.

Activity	Duration	Immediate predecessors
A	2	None
B	4	None
C	5	A
D	3	B
E	2	C and D

(a) Find the earliest start and finish of every activity. [4 marks]

C follows A; D follows B.

E starts at $\max(7, 7) = 7$.

Answer: A: 0-2, B: 0-4, C: 2-7, D: 4-7, E: 7-9.

(b) State all critical paths and the minimum completion. [3 marks]

Both branches before E take 7 days; E adds 2.

Answer: A-C-E and B-D-E;9 days.

2. Use the science-display project. [6 marks]

Preparing a science display; all durations are in days.

Activity	Duration	Immediate predecessors
A	2	None
B	4	None
C	5	A
D	3	B
E	2	C and D

(a) Find the latest starts of A, B, C, D, E for a day-9 completion. [4 marks]

E latest 7; C latest $7-5=2$; D latest $7-3=4$.

A latest $2-2=0$; B latest $4-4=0$.

Answer: A: 0; B: 0; C: 2; D: 4; E: 7 days.

(b) Explain why all activities have zero float even though not all lie on one single route. [2 marks]

Each activity belongs to at least one controlling start-to-finish route.

Zero float does not imply a single serial chain.

Answer: There are two critical branches.

3. Each change to the science-display project is considered separately. [7 marks]

Preparing a science display; all durations are in days.

Activity	Duration	Immediate predecessors
A	2	None
B	4	None
C	5	A
D	3	B
E	2	C and D

(a) Reduce C by 2 days. Find the finish. [2 marks]

A-C-E becomes $2+3+2=7$; B-D-E stays 9.

Answer: 9 days.

(b) Reduce C by 2 days and D by 1 day. Find the finish. [3 marks]

BeforeE the branches take $2+3=5$ and $4+2=6$.

E starts 6 and ends 8.

Answer: 8 days.

(c) Instead reduce only E by 1 day. Find the finish. [2 marks]

E is shared by both critical routes, so both route totals fall to 8.

Answer: 8 days.

4. Only A and B in the science-display project share one crew. C, D, E have separate available crews. [6 marks]

Preparing a science display; all durations are in days.

Activity	Duration	Immediate predecessors
A	2	None
B	4	None
C	5	A
D	3	B
E	2	C and D

(a) If A is done before B, find the earliest completion. [3 marks]

A: 0-2, C: 2-7; B: 2-6, D: 6-9; E: 9-11.

Answer: 11 days.

(b) If B is done before A, find the earliest completion. Recommend the better order. [3 marks]

B: 0-4, D: 4-7; A: 4-6, C: 6-11; E: 11-13.

The order of the shared-crew tasks affects the feasible finish.

Answer: 13 days; choose A before B.

5. Ali cannot perform job 2 in the cost table. [5 marks]

Costs ($); an unavailable pairing cannot be used.

Worker	1	2	3
Ali	9	Unavailable	7
Bea	6	4	3
Chen	5	8	1

(a) Find a minimum feasible assignment and total. [3 marks]

Exclude allocations containing Ali/job 2.

Remaining totals 14,20,21,16 give minimum 14.

Answer: Ali→1, Bea→2, Chen→3;$14.

(b) A software model replaces the unavailable entry by 1000. Justify this substitution for these nonnegative costs. [2 marks]

The numerical bound is tied to a feasible alternative and nonnegative remaining costs.

Answer: Any selected 1000 entry produces a total at least 1000, worse than the known feasible 14.

6. Minimise the displayed cost table. [5 marks]

Assignment costs ($).

Worker	1	2	3
A	4	4	9
B	4	4	9
C	9	9	1

(a) Give two different optimal assignments and their costs. [3 marks]

$4+4+1=9$ in either case.

Answer: A→1, B→2, C→3 and A→2, B→1, C→3; both $9.

(b) Prove that no assignment can cost less. [2 marks]

This is a valid lower bound and matching feasible total.

Answer: The sum of row minima is $4+4+1=9$ and a feasible assignment attains it.

7. The table gives net profits in $100 units, including losses. [6 marks]

Net profit ($100 units); negative entries are losses.

Team	1	2	3
A	5	-2	3
B	4	6	-1
C	-3	2	7

(a) Transform it to a minimisation problem using $K=7$. [3 marks]

Use $7-\text{profit}$ even for negative entries;$7-(-2)=9$.

Answer: $\begin{bmatrix} 2 & 9 & 4 \\ 3 & 1 & 8 \\ 10 & 5 & 0 \end{bmatrix}$.

(b) Find the best allocation and total profit. [3 marks]

The jointly feasible row maxima are 5,6,7.

Their sum 18 table units equals $1,800.

Answer: A→1, B→2, C→3;$1,800.

8. Three workers cover two jobs; free idling is allowed. [6 marks]

Costs ($); an idle worker costs $0.

Worker	1	2
A	6	4
B	5	3
C	2	8

(a) Add the required dummy structure and find the optimum. [3 marks]

The real-job minima 2 and 3 are attained by different workers.

Answer: One zero dummy-job column; C→1, B→2, A idle; cost $5.

(b) The cost of leaving A idle is revised to $4; B and C may still idle for free. Find the new optimum. [3 marks]

Original allocation now costs $2+3+4=9$.

Using C→1, A→2 costs $2+4+0=6$; all other feasible pairings cost at least 6.

Answer: C→1, A→2, B idle; cost $6.

9. Using the unfilled-job penalty table, a manager reports a \$7 assignment by A→2 and B→3. [5 marks]

Costs (\$), including a permitted unfilled-job penalty row.

Resource	1	2	3
A	6	4	8
B	5	7	3
Unfilled	10	2	7

(a) Identify the missing term and correct this allocation's cost. [2 marks]

The dummy row must contribute its selected entry.

Answer: Job 1's unfilled penalty \$10 is missing; total $4+3+10=\$17$.

(b) Find the better allocation and state precisely what remains undone. [3 marks]

$6+3+2=11$.

A dummy represents the uncompleted job 2, not an extra worker.

Answer: A→1, B→3, job 2 unfilled; total \$11.

10. A council's model predicts a \$14 assignment and a 9-day project. It uses last year's costs and assumes activities can overlap freely. [5 marks]

(a) State two checks needed before adopting the recommendation. [2 marks]

Both input reliability and feasibility affect the recommendation.

Answer: For example update costs and check resource availability for parallel tasks.

(b) Explain how to test sensitivity to a likely delay without claiming the old critical path must remain critical. [3 marks]

A previously non-critical branch can become controlling.

Report the tested change and revised schedule.

Answer: Increase the affected duration, repeat the scans, and compare the new finish and zero-float routes.