Networks, scheduling and assignment - Test 2

10 questions | 59 marks | 65 minutes

Develop scheduling calculations and compare minimum-cost, maximum-profit and rectangular assignments.

Equivalent correct methods earn the corresponding marks.

1. Use the exhibition-project table. [6 marks]

A community exhibition project; durations are in hours.

Activity	Duration	Immediate predecessors
A	3	None
B	5	None
C	4	A
D	2	A
E	6	B
F	3	C and D
G	2	E and F

(a) Draw an activity-on-arrow network with the stated dependencies. Explain where a zero-duration logical connection may be needed. [3 marks]

A and B start independently.

C and D both follow A; E follows B; F needs C, D; G needs E, F.

Use dummy arrows if required to express these joins without extra predecessors.

Answer: Any acyclic equivalent network is acceptable; merges must enforce C and D before F and E and F before G.

(b) Find the earliest start of F and G. [3 marks]

C finishes 7, D finishes 5, so F starts 7 and finishes 10.

E finishes 11, so G starts $\max(10, 11) = 11$.

Answer: F:7 hours; G:11 hours.

2. Use the exhibition project again. [7 marks]

A community exhibition project; durations are in hours.

Activity	Duration	Immediate predecessors
A	3	None
B	5	None
C	4	A
D	2	A
E	6	B
F	3	C and D
G	2	E and F

(a) Find the minimum completion and a critical path. [3 marks]

The B-E-G route takes $5+6+2=13$ hours.

$\mathrm{A}-\mathrm{C}-\mathrm{F}-\mathrm{G}=12$; $\mathrm{A}-\mathrm{D}-\mathrm{F}-\mathrm{G}=10$.

Answer: 13 hours; B-E-G.

(b) Find the latest starts of A, C, D, F. [4 marks]

G starts latest 11; F latest 8; E latest 5.

C latest $8-4=4$; D latest $8-2=6$; A latest $\min(4,6)-3=1$.

Answer: A: 1 hour; C: 4 hours; D: 6 hours; F: 8 hours.

3. In the exhibition project, assess each separate change. [6 marks]

A community exhibition project; durations are in hours.

Activity	Duration	Immediate predecessors
A	3	None
B	5	None
C	4	A
D	2	A
E	6	B
F	3	C and D
G	2	E and F

(a) Increase F from 3 to 5 hours. [3 marks]

F finishes $\max(7, 5) + 5 = 12$.

G starts $\max(\text{E } 11, \text{F } 12) = 12$ and finishes 14.

Answer: Completion 14 hours.

(b) Reduce E from 6 to 4 hours. [3 marks]

E finishes 9; F still finishes 10.

G waits for F and ends 12.

Answer: Completion 12 hours, not 11.

4. Use the exhibition project to discuss float. [6 marks]

A community exhibition project; durations are in hours.

Activity	Duration	Immediate predecessors
A	3	None
B	5	None
C	4	A
D	2	A
E	6	B
F	3	C and D
G	2	E and F

(a) Find the floats of A, C, D, F. [3 marks]

Subtract earliest starts 0,3,3,7 from latest starts 1,4,6,8.

Answer: A: 1 hour; C: 1 hour; D: 3 hours; F: 1 hour.

(b) A and C are each extended by 1 hour. Find the new project finish and explain the shared allowance. [3 marks]

A-C-F-G rises from 12 to 14.

The original one-hour path allowance cannot be used twice.

Answer: 14 hours.

5. A different project has two independent routes lasting 10 and 14 hours, followed by a common 3-hour final activity. [5 marks]

(a) Find the earliest finish and the maximum extension of the shorter route that preserves it. [3 marks]

The final task starts at 14.

The shorter route can reach 14 without changing that start.

Answer: 17 hours;4 hours extension.

(b) The shorter route needs a machine also used by the longer route. Explain why the answer may need revision. [2 marks]

The route-duration comparison assumes the two routes can run independently.

Answer: Resource conflict may prevent their assumed overlap.

6. Minimise the repair times in the table. [6 marks]

Time in hours for one worker to complete one separate repair.

Worker	X	Y	Z
A	7	4	9
B	6	8	5
C	9	3	7

(a) Show row reduction. [3 marks]

Subtract row minima 4,5,3.

Answer: $\begin{bmatrix} 3 & 0 & 5 \\ 1 & 3 & 0 \\ 6 & 0 & 4 \end{bmatrix}$.

(b) Find the optimum assignment and original total. [3 marks]

Totals of all six complete assignments are 22,15,17,18,18,26.

Choose 15.

Answer: A→X, B→Z, C→Y;15 hours.

7. Use the four-worker cost table. [6 marks]

Assignment costs ($). Each worker does one job.

Worker	1	2	3	4
A	8	3	7	6
B	5	9	4	8
C	6	7	2	5
D	7	4	8	3

(a) Find a minimum complete assignment. [4 marks]

Original total $3+5+2+3=13$.

Exhaustive enumeration or Hungarian reduction confirms no smaller complete total.

Answer: A→2, B→1, C→3, D→4;$13.

(b) Explain why the sum of the four row minima is not immediately a feasible assignment. [2 marks]

Row-minimum sum $3+4+2+3=12$ violates the distinct-column rule.

Answer: B and C both choose job 3.

8. The contract-profit table uses $1,000 units. [6 marks]

Contract profit in $1,000 units.

Team	1	2	3	4
A	14	18	12	11
B	16	13	15	12
C	10	16	14	17
D	15	11	18	14

(a) Find the maximum profit and corresponding pairing. [4 marks]

Each row maximum occurs in a different column: $18 + 16 + 17 + 18 = 69$.

Answer: A→2, B→1, C→4, D→3;$69,000.

(b) Explain why taking the row maxima is sufficient for this particular table but not a general rule. [2 marks]

No assignment can exceed the sum of row maxima, and this one attains it.

Answer: They happen to be jointly feasible; in another table row maxima could conflict.

9. Use the two-worker table, where leaving one job unfilled is permitted at its stated penalty. [6 marks]

Costs ($), including a permitted unfilled-job penalty row.

Resource	1	2	3
A	6	4	8
B	5	7	3
Unfilled	10	2	7

(a) Find the optimum and identify the unfilled job. [4 marks]

The six totals are 20,11,16,17,15,25.

The minimum includes the $2 penalty for job 2.

Answer: A→1, B→3, job 2 unfilled; total $11.

(b) Would the same dummy row be valid if every job had to be physically completed? Explain. [2 marks]

The dummy represents omission, not a real person.

Answer: No; an extra worker or revised capacity model is needed.

10. A dispatcher compares two feasible assignments. Allocation U gives driving times 2,3,12 hours. Allocation V gives 5,6,7 hours. All drivers leave together. [5 marks]

(a) Choose the assignment minimising total driving hours. [2 marks]

$2+3+12=17; 5+6+7=18.$

Answer: U:17 hours total, versus 18 for V.

(b) Choose the assignment minimising latest arrival, and explain why this differs. [3 marks]

Latest arrival uses a maximum rather than a sum.

The objective must be agreed before declaring one assignment best.

Answer: V: latest 7 hours, versus 12 for U.