Hungarian algorithm

6 questions | 17 marks

Equivalent correct methods earn the corresponding marks.

1. For the matrix rows $\begin{bmatrix} 8 & 5 & 7 \\ 6 & 9 & 4 \\ 3 & 7 & 6 \end{bmatrix}$, perform row reduction. [3 marks]

The row minima are 5,4 and 3.

Subtract each minimum from every entry in its row.

Answer: $\begin{bmatrix} 3 & 0 & 2 \\ 2 & 5 & 0 \\ 0 & 4 & 3 \end{bmatrix}$.

2. For the reduced matrix $\begin{bmatrix} 3 & 0 & 2 \\ 2 & 5 & 0 \\ 0 & 4 & 3 \end{bmatrix}$, is a further column reduction needed? Find a complete zero assignment. [2 marks]

The three zeros occupy distinct rows and columns.

Answer: No; each column minimum is 0. Choose row 1→column 2, row 2→column 3, row 3→column 1.

3. Use the Amy/Ben/Cal table to show why two independent zeros are not enough for a complete allocation. [3 marks]

Times in hours for one person to perform one task.

Person	X	Y	Z
Amy	4	1	3
Ben	2	0	5
Cal	3	2	2

After reductions, Amy and Ben initially both rely on column Y for their zero.

Cal can use another column, but Amy and Ben still conflict.

Answer: There are three people/jobs, so three independent zeros are required.

4. Zeros are covered by row 3 and column 2 in $\begin{bmatrix} 2 & 0 & 2 \\ 1 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix}$. Carry out the adjustment. [4 marks]

The smallest uncovered entry is 1.

Subtract 1 at $(1,1),(1,3),(2,1),(2,3)$.

Add 1 at the double-covered entry $(3,2)$; other covered entries stay unchanged.

Answer: $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 4 \\ 0 & 1 & 0 \end{bmatrix}$.

5. For the original Amy/Ben/Cal table, compare the algorithm's 5-hour assignment with Amy→X, Ben→Y, Cal→Z. [2 marks]

Times in hours for one person to perform one task.

Person	X	Y	Z
Amy	4	1	3
Ben	2	0	5
Cal	3	2	2

The zero in Ben/Y was originally zero but cannot alone determine the optimum.

Use original entries for both totals.

Answer: The alternative is $4+0+2=6$ hours, one hour slower.

6. A student draws one line through every row, covering all zeros in any 3×3 reduced matrix, and immediately stops. Explain the error. [3 marks]

Three row lines always cover everything, even when the zeros cannot form a feasible allocation.

An arbitrary cover does not establish optimality.

Answer: The algorithm needs the minimum number of covering lines or an explicit complete independent-zero assignment.