Financial mathematics - practice test 6

10 questions | 60 marks | 75 minutes

A harder synthesis paper: reverse models, changing conditions, cash-flow cost rates and refinancing.

Equivalent correct methods earn the corresponding marks.

1. AUD 7,500 grows to AUD 9,000 in 5 years under a constant annual compound model. [6 marks]

(a) Find the effective annual rate. [3 marks]

Solve $(1+i)^5 = \frac{9000}{7500}$.

Answer: 3.7137%

(b) Find the monthly rate equivalent to this annual growth. [3 marks]

The same total growth spans 60 months; take the 60th root.

Answer: 0.3043%

2. Two accounts accept AUD 12,000 for one year. A quotes 6% nominal compounded quarterly with an AUD 40 fee at year end. B quotes 5.8% nominal compounded monthly with no fee. [6 marks]

(a) Find both net ending balances. [3 marks]

Apply the appropriate growth factor and then the stated fee.

Answer: A:AUD 12,696.36; B:AUD 12,714.80.

(b) Find the starting balance at which the two one-year ending balances are equal. [3 marks]

Solve $P(g_A - g_B) = 40$.

This crossover applies only to the one-year model and listed fee.

Answer: AUD 22,264.22

3. A target has today's purchasing power AUD 30,000. It is required in 8 years; inflation is 2.5% annually and the savings account earns 4.5% annually. [6 marks]

(a) Find the future-dollar goal. [3 marks]

Grow the price target at inflation, not at the investment rate.

Answer: AUD 36,552.09

(b) Find the equal year-end deposit from zero that meets it. [3 marks]

Use the 8-year savings factor at 4.5%.

Answer: AUD 3,896.81

4. AUD 5,000 is available now, and 10 equal deposits will be made at the beginnings of years 1 through 10-that is, times 0 through 9. The balance at time 10 must be AUD 25,000. Interest is 4% annually. [6 marks]

(a) Calculate the required deposit R. [3 marks]

The initial 5000 grows for 10 years.

Each of the 10 deposits earns one more period than an ordinary annuity.

Answer: AUD 1,409.44

(b) Explain why the first deposit is not included in the AUD 5,000 initial balance in this model. [3 marks]

Account for each cash flow once; here those are separate initial contributions.

Answer: The problem specifies AUD 5,000 already available plus an additional equal deposit at time 0.

5. A fund starts at AUD 30,000. Its supplied annual returns are 5%, -2% and 6%. Gross year-end contributions are AUD 4,000 each year, with a stated 15% deduction. Fixed year-end fees are AUD 100, AUD 120 and AUD 140 respectively. [6 marks]

Fund scenario

Year	Return	Gross contribution (AUD)	Fixed fee (AUD)
1	5%	4000	100
2	-2%	4000	120
3	6%	4000	140

(a) Find the net contribution and year 1 closing balance. [3 marks]

Net contribution is $4000 \times 0.85 = 3400$.

$B_1 = 30000 \times 1.05 + 3400 - 100 = 34800$.

Answer: AUD 3,400.00; AUD 34,800.00.

(b) Find the final balance and explain why a single average-rate annuity calculation is not the stated model. [3 marks]

Use the recurrence year by year.

A constant-rate annuity would impose different assumptions.

Answer: AUD 42,887.04; returns and fees change between years.

6. An account starts with AUD 18,000 and pays AUD 500 at the end of each of 12 months at zero interest. It then earns 0.4% monthly for another 12 months while keeping the same withdrawals. [6 marks]

(a) Find the balance after the zero-interest stage. [3 marks]

$18000 - 12 \times 500 = 12000$.

Answer: AUD 12,000.00

(b) Find the balance after all 24 withdrawals. [3 marks]

Use the time 12 balance as the new principal.

For stage 2 calculate $12000(1.004)^{12} - 500s_{12}(0.004)$.

Answer: AUD 6,455.07

7. A balance earns -4% during a year, then a fee of 1% of the remaining balance is deducted. Prices fall 1% during the same year. [6 marks]

(a) Find the net nominal return. [3 marks]

Net factor is $0.96 \times 0.99 = 0.9504$.

Return is $0.9504 - 1 = -0.0496$.

Answer: -4.96%

(b) Find the real return and explain whether falling prices entirely offset the loss. [3 marks]

Real factor is $\frac{0.9504}{0.99} = 0.96$.

The real return is -4%.

Answer: -4.0000%; no, purchasing power still falls.

8. A business has an interest-only debt of AUD 50,000 at 6% annually due in 5 years. It already holds AUD 10,000 in a sinking fund earning 4% annually and will add equal year-end deposits. [6 marks]

(a) Find the required sinking deposit. [3 marks]

Grow the initial fund, subtract from 50000, then divide by $s_5(0.04)$.

Answer: AUD 6,985.08

(b) Find annual combined cash outflow, then separate total loan interest from total new fund deposits. [3 marks]

Interest is $50000 \times 0.06 = 3000$ each year.

Fund deposits build a separate asset; they are not all interest cost.

Answer: Annual:AUD 9,985.08; loan interest:AUD 15,000.00; new deposits:AUD 34,925.42.

9. A loan currently owes AUD 20,000 and has 24 months remaining. Keeping it costs 24 month-end repayments of AUD 900. Refinancing finances an AUD 300 fee into the new debt; the new nominal rate is 6% annually, compounded monthly, over 24 months. There are no other costs. [6 marks]

(a) Find the new monthly repayment. [3 marks]

New principal is 20300; monthly rate 0.005.

Use $R = \frac{20300}{a_{24}(0.005)}$.

Answer: AUD 899.71

(b) Compare the total future cash outflows and state a relevant model limitation. [3 marks]

The financed fee is already included in new principal; do not add it again.

The comparison assumes the stated full term and excludes any other exit or account costs.

Answer: Keep:AUD 21,600.00; refinance:AUD 21,593.00.

10. A loan starts at AUD 12,000 with 0.5% monthly interest and AUD 500 month-end payments. Model A has a constant AUD 2,000 offset. Model B instead uses that AUD 2,000 to reduce principal immediately and has no offset. Use no negative interest and unrounded balances. [6 marks]

(a) Find both loan balances after 6 payments. [3 marks]

A uses interest on $B - 2000$; B starts with principal 10000.

Apply each recurrence for 6 months.

Answer: A:AUD 9,266.02; B:AUD 7,266.02.

(b) Compare net debt after allowing for A’s offset asset, and explain why the displayed loan balances differ. [3 marks]

As long as the offset remains fully usable, net debt follows the same recurrence as B.

A’s displayed debt is 2000 higher because it still has the separate 2000 asset.

Answer: A net debt:AUD 7,266.02; B net debt:AUD 7,266.02. They are equal in this model.