Marking guide | Financial mathematics

Financial mathematics - practice test 5

10 questions | 60 marks | 75 minutes

Use multi-stage savings, retirement and loan models with full supplied assumptions.

Equivalent correct methods earn the corresponding marks.

1. A future amount of AUD 25,000 is needed in 7 years. An account earns 0.45% monthly. [6 marks]

(a) State the number of compounding periods. [3 marks]

$7 \times 12 = 84$.

Answer: 84 months

(b) Find the single deposit required now. [3 marks]

$P = \frac{25000}{(1.0045)^{84}}$.

Answer: AUD 17,145.29

2. AUD 8,000 grows for 3 years at 4% effective annually, then 2 years at 5% effective annually. [6 marks]

(a) Find the final balance. [3 marks]

Multiply the three-year and two-year growth factors.

Answer: AUD 9,921.30

(b) Find the constant annual effective rate that gives the same five-year result. [3 marks]

Let $(1+i)^5 = 1.04^3 \times 1.05^2$.

Take the fifth root then subtract 1.

Answer: 4.3989%

3. A saver deposits AUD 500 at each year end for 5 years, then AUD 800 at each year end for another 5 years. The account earns 4% annually. [6 marks]

(a) Find the balance immediately after year 5. [3 marks]

Use $500s_{5}(0.04)$.

Answer: AUD 2,708.16

(b) Find the balance at time 10. [3 marks]

Grow the first-stage balance for 5 more years.

Add the future value of the five AUD 800 deposits.

Answer: AUD 7,627.95

4. An initial AUD 1,500 plus monthly end deposits of AUD 200 earns 0.3% monthly. The target is AUD 10,000. [6 marks]

(a) Find the first whole month reaching the target. [3 marks]

Rearrange $B_n = (1500 + \frac{200}{0.003})(1.003)^n - \frac{200}{0.003}$.

Solve the inequality and round the month count upwards.

Answer: 40

(b) Calculate the interest earned by that month. [3 marks]

Subtract initial savings and all completed deposits from the model balance.

Answer: AUD 677.23

5. In this model, salary starts at AUD 50,000 and grows 3% each year. Gross contributions are 12% of that year's salary, with a 15% deduction. A fund starts at AUD 10,000 and earns 4% annually. Contributions occur at each of the next 3 year ends; the first salary is AUD 50,000. [6 marks]

(a) Find the three net contributions. [3 marks]

Net contribution is $\text{salary} \times 0.12 \times 0.85$.

Use salaries 50000,51500,53045.

Answer: AUD 5,100.00, AUD 5,253.00, AUD 5,410.59.

(b) Find the balance after year 3. [3 marks]

Multiply the previous balance by 1.04, then add that year's net contribution.

Answer: AUD 27,638.51

6. A retirement account must pay AUD 10,000 at the end of year 1, AUD 10,300 at year 2 and AUD 10,609 at year 3. It earns 5% annually. [6 marks]

(a) Find the present value of these unequal withdrawals. [3 marks]

Discount each specified payment separately.

Do not use an equal-payment annuity factor unchanged.

Answer: AUD 28,030.67

(b) Verify that the calculated starting balance is exhausted after the third withdrawal. [3 marks]

Apply interest then deduct 10000,10300,10609 in sequence.

Answer: The exact model leaves zero; final-cent display rounding may leave a few cents.

7. Two fee plans apply to a single AUD 10,000 deposit for 10 years at 5% before fees. Plan A charges AUD 80 at each year end. B charges 1% of the post-interest balance each year. [6 marks]

(a) Find each final balance. [3 marks]

A uses a fixed annual withdrawal.

B uses the annual multiplier 1.05×0.99.

Answer: A:AUD 15,282.71; B:AUD 14,731.43.

(b) Choose the higher model balance and explain why subtracting 1 percentage point from 5% is not exact for B. [3 marks]

The percentage fee is applied after the interest, so multipliers must be multiplied.

Answer: Plan A has the higher balance; B's exact multiplier is 1.0395, not 1.04.

8. Equipment costing AUD 40,000 today must be replaced after 5 years. Its replacement price rises 2% annually. A sinking fund earns 3.5% annually with equal year-end deposits. [6 marks]

(a) Find the replacement target. [3 marks]

Inflate the current price by 1.02^5.

Answer: AUD 44,163.23

(b) Find the annual sinking-fund deposit. [3 marks]

Divide the future-dollar target by $s_5(0.035)$.

Answer: AUD 8,235.62

9. AUD 15,000 is owed at 0.5% monthly. No repayments are made for the next 3 months, and interest is capitalised. The resulting debt is then cleared over 36 equal month-end repayments at the same rate. [6 marks]

(a) Find the debt after the holiday. [3 marks]

No cash is paid, so use compound growth for 3 months.

Answer: AUD 15,226.13

(b) Find the new monthly repayment. [3 marks]

Use the enlarged debt as P in $R = \frac{P}{a_{\overline{36}|}(0.005)}$.

Answer: AUD 463.21

10. Compare one-year saving schedules with zero initial balance and a fixed effective annual rate of 6%. A deposits AUD 100 at each month end; B deposits AUD 1,200 at year end. Use the exactly equivalent monthly rate for A. [6 marks]

(a) Find both balances at year end. [3 marks]

For A, $i = 1.06^{\frac{1}{12}} - 1$.

B’s only deposit earns no interest before the measurement date.

Answer: A:AUD 1,232.65; B:AUD 1,200.00.

(b) Explain why the result is not caused by a different annual interest rate. [3 marks]

The deposit timing differs even though total deposits equal AUD 1,200.

Answer: Both use the same 6% effective annual rate; A invests some money earlier.