Financial mathematics - practice test 4

10 questions | 60 marks | 75 minutes

Explain modelling errors and adapt recurrences to missed payments, changing rates and offsets.

Equivalent correct methods earn the corresponding marks.

1. A saver says that 5% annual compound interest grows an amount by exactly 50% after 10 years because $5 \times 10 = 50$. [6 marks]

(a) Find the actual 10-year growth factor. [3 marks]

Calculate $1.05^{10} \approx 1.628895$.

Answer: 1.628895

(b) Find the first whole year in which the balance is at least double. [3 marks]

Solve $1.05^n \geq 2$.

$n \geq \frac{\ln(2)}{\ln(1.05)} \approx 14.2067$; use 15 years.

Answer: 15 years

2. A spreadsheet uses 0.06 as a monthly rate for a quote of 6% nominal annually compounded monthly. [6 marks]

(a) State and correct the error. [3 marks]

The nominal quote must be divided by 12.

Answer: It used 6% per month instead of 0.5%; use $\frac{0.06}{12} = 0.005$.

(b) Find the correct one-year balance of AUD 2,000. [3 marks]

Use $2000(1.005)^{12}$.

Answer: AUD 2,123.36

3. An account starts with AUD 1,000 and earns 4% annually. Deposit AUD 300 at the ends of years 1 and 3, but nothing at year 2. [6 marks]

(a) Find the balance after year 2. [3 marks]

Year 1 balance is $1000 \times 1.04 + 300 = 1340$.

Year 2: $1340 \times 1.04 = 1393.60$.

Answer: AUD 1,393.60

(b) Find the year 3 balance and explain why the usual three-payment annuity formula is unsuitable without an adjustment. [3 marks]

Grow the year 2 balance by 1.04 then add 300.

The ordinary formula assumes every stated deposit is made.

Answer: AUD 1,749.34; the year 2 payment is absent.

4. An account starts with AUD 2,000 and receives AUD 700 at each year end, earning 4%. Its goal is AUD 8,000. [6 marks]

(a) Find the first whole year reaching the goal. [3 marks]

Solve $2000(1.04)^n + 700s_n(0.04) \geq 8000$.

Using logarithms gives $n \approx 6.8399$, so test 7.

Answer: 7 years

(b) Check the balances at years 6 and 7. [3 marks]

Substitute 6 and 7 into the full balance model.

Answer: Year 6:AUD 7,173.72; year 7:AUD 8,160.67.

5. Two two-year scenarios start at AUD 10,000 and add AUD 2,000 at each year end. Scenario A has returns of $+10\%$, then -10%. Scenario B has returns of -10%, then $+10\%$. [6 marks]

(a) Find both final balances. [3 marks]

Apply each return to the opening balance, then add 2000.

Answer: A:AUD 13,700.00; B:AUD 14,100.00.

(b) Explain why the same two returns give different outcomes. [3 marks]

The original principal receives the same combined 0.99 factor.

Contributions make the order of returns matter.

Answer: The first contribution is exposed to the second year’s return, which differs between the scenarios.

6. AUD 30,000 earns 5% annually and pays AUD 4,000 at each year end. [6 marks]

(a) Find the first closing balance and the first principal reduction. [3 marks]

Interest is 1500; closing balance is $30000 + 1500 - 4000 = 27500$.

Answer: AUD 27,500.00; principal reduction AUD 2,500.00.

(b) Find the number of withdrawals needed to exhaust the balance if the final withdrawal is reduced. [3 marks]

This uses the same decreasing-balance recurrence as a loan.

Find the first period for which the next full withdrawal would exceed the available balance.

Answer: 10

7. A savings account starts at AUD 5,000, earns 3% annually and receives AUD 800 each year end. The year-end fixed fees are AUD 40, AUD 60 and AUD 80 in years 1-3. [6 marks]

Fees supplied

Year	Fee (AUD)
1	40
2	60
3	80

(a) Let B_k be the closing balance in year k and F_k that year's fee. Write the recurrence. [3 marks]

Interest is added to the previous year's closing balance, then the current contribution and fee are applied.

Answer: $B_k = 1.03B_{k-1} + 800 - F_k$

(b) Find the balance after year 3. [3 marks]

Apply the changing fee in each year; do not use one constant annuity cash flow.

Answer: AUD 7,752.12

8. A sinking fund needs AUD 20,000 at time 6. It receives equal year-end deposits R for the first 3 years at 4%. The savings rate then becomes 2% for the final 3 years. [6 marks]

(a) If R was originally chosen assuming 4% throughout, calculate it and the time 3 balance. [3 marks]

$R = \frac{20000}{s_6(0.04)}$.

$B_3 = R\, s_3(0.04)$.

Answer: R:AUD 3,015.24; B_3:AUD 9,412.37.

(b) Find a revised equal deposit for the final 3 years that still reaches AUD 20,000. [3 marks]

Grow the time 3 balance at 2% for 3 years.

Divide the remaining target by $s_3(0.02)$.

Answer: AUD 3,271.31

9. A loan balance is AUD 4,000 at 1% monthly, but its monthly repayment is only AUD 30. [6 marks]

(a) Find the first two closing balances. [3 marks]

$B_1 = 4000 \times 1.01 - 30 = 4010.$

$B_2 = 4010 \times 1.01 - 30 = 4020.10.$

Answer: AUD 4,010.00 and AUD 4,020.10

(b) Explain why a payoff-time solver cannot give a valid positive finite term under these assumptions. [3 marks]

$R = 30$ is less than $Pi = 40.$

The repayment condition needed for a finite decreasing-balance payoff is not met.

Answer: Each payment is below even the initial interest; the balance and interest grow.

10. A loan owes AUD 50,000; an offset holds AUD 8,000. Monthly interest is 0.5%, followed by an AUD 1,000 repayment. No interest is paid on the offset. [6 marks]

(a) Find the first month's interest and closing loan balance. [3 marks]

Interest is $(50000 - 8000) \times 0.005 = 210.$

Closing debt is $50000 + 210 - 1000 = 49210.$

Answer: AUD 210.00; AUD 49,210.00.

(b) Find the debt minus offset after this payment, and explain what the offset does not change. [3 marks]

Net debt is $49210 - 8000 = 41210.$

The offset remains a separate asset.

Answer: AUD 41,210.00; it does not itself subtract AUD 8,000 from the displayed loan principal.