Annuities providing an income

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. Find the capital required for 8 end-of-year withdrawals of AUD 3,000 at 3%. [3 marks]

Use $P = R\,a_{\overline{n}|}$.

$P = \frac{3000(1 - 1.03^{-8})}{0.03}$.

Answer: AUD 21,059.08

2. AUD 12,000 must provide 24 equal month-end withdrawals at zero interest. Find each withdrawal. [3 marks]

At zero interest, total withdrawals equal starting capital.

$R = \frac{12000}{24} = 500$.

Answer: AUD 500.00

3. An account starts with AUD 40,000, earns 4% annually, and pays AUD 4,000 at each year end. Find the balance after 5 withdrawals. [3 marks]

Initial money grows for 5 years.

Subtract the accumulated value of the five withdrawals.

$B_5 = 40000(1.04)^5 - \frac{4000((1.04)^5 - 1)}{0.04}$.

Answer: AUD 27,000.83

4. Compare the starting capital for five AUD 2,000 withdrawals at 5% when they occur at year ends and at year beginnings. [3 marks]

Year-beginning withdrawals happen one period earlier.

Their present value is 1.05 times the ordinary-annuity present value.

Answer: End: AUD 8,658.95; beginning: AUD 9,091.90.

5. A retirement model has AUD 100,000 earning 4% annually and pays AUD 6,000 at year end. Find the balance after the first year and explain why it falls. [3 marks]

Interest is $100000 \times 0.04 = 4000$.

After withdrawal the balance is $100000 + 4000 - 6000 = 98000$.

The withdrawal exceeds that year's interest by AUD 2,000.

Answer: AUD 98,000.00

6. An exactly funded annuity has 6 withdrawals of AUD 2,500 still to come, starting next year, at 4%. Find its present balance. [3 marks]

At the current date, the remaining payments form a new ordinary annuity.

Balance is $\frac{2500(1 - 1.04^{-6})}{0.04}$.

Answer: AUD 13,105.34