Statistics and regression - Practice test 3

10 questions | 51 marks | 70 minutes

Choose and assess exponential models alongside linear summaries and normal calculations.

Equivalent correct methods earn the corresponding marks.

1. A population count is 120,180,270,405 at times 0,1,2,3 days. [5 marks]

Paired measurements

Days	Count
0	120
1	180
2	270
3	405

(a) Explain why an exponential model is suggested. [2 marks]

Differences are not constant, but ratios are.

Answer: Each successive response is multiplied by 1.5.

(b) Write the model and predict at day 4. [3 marks]

The initial count is 120.

$120 \times 1.5^4 = 607.5$.

Answer: $y = 120(1.5)^t$; 607.5.

2. A decreasing model is $y = 800(0.85)^t$. [5 marks]

(a) State the percentage decrease per time unit. [2 marks]

0.85 retains 85% of the preceding modelled amount.

Answer: 15%.

(b) Calculate the decrease over four time units as a percentage. [3 marks]

The four-unit multiplier is 0.85^4.

Subtract from 1, then multiply by 100.

Answer: 47.8%.

3. Technology gives $\ln(\hat{y}) = 4.1 - 0.12t$. [5 marks]

(a) Express the result in original response units. [3 marks]

Exponentiate both sides.

The initial scale is $e^{4.1}$, not 4.1.

Answer: $\hat{y} \approx 60.3403e^{-0.12t}$.

(b) Predict y at $t = 5$. [2 marks]

Substitute into $e^{4.1 - 0.12 \times 5}$.

Answer: 33.12.

4. A class fits a log model only after removing responses 0 and -2 from the original table. [5 marks]

(a) Why do these responses prevent direct real-log fitting? [2 marks]

A real-log transformation requires positive responses.

Answer: $\ln(y)$ is undefined for $y \leq 0$.

(b) Why is silently deleting them not a satisfactory solution? [3 marks]

Investigate measurement validity and a suitable alternative model, and disclose treatment.

Answer: Deletion changes the sample and may bias the conclusion.

5. For a straight-line fit to $\ln(y), r = -0.96$. [5 marks]

(a) Find the coefficient of determination. [2 marks]

Square -0.96.

Answer: 0.9216.

(b) Give its correct scale-specific interpretation. [3 marks]

The response used in this fit is $\ln(y)$.

Do not replace log-response variation with original-response variation.

Answer: 92.16% of the variation in $\ln(y)$ is accounted for by its fitted linear relationship with x.

6. At $x = 1, 2, 3, 4$, residuals from a proposed linear model are $3, -2, -3, 2$. [5 marks]

Paired measurements

x	Residual
1	3
2	-2
3	-3
4	2

(a) Sketch the residual plot and describe its pattern. [3 marks]

Plot the supplied pairs on residual-versus-x axes.

Compare with a zero reference line.

Answer: Positive ends and negative middle suggest curvature.

(b) What follow-up is appropriate? [2 marks]

Do not accept a linear form just because a correlation is large.

Answer: Investigate a nonlinear model and compare its residual behaviour in the context.

7. A regression of cost C on output q has slope 4.8 dollars per item and passes through $(50, 310)$. [5 marks]

(a) Find its equation. [3 marks]

$a = 310 - 4.8 \times 50 = 70.$

Answer: $\hat{C} = 70 + 4.8q$.

(b) Interpret the intercept cautiously. [2 marks]

A fitted intercept needs contextual justification.

Answer: The model predicts 70 dollars at zero output, which may represent a base cost if zero output is meaningful.

8. A model was fitted to a culture for only the first four days. [5 marks]

(a) Why is predicting its size after 100 days risky even if the model fits perfectly? [3 marks]

A short-term exponential pattern need not persist indefinitely.

Answer: Resources and growth conditions can change; the prediction is far outside the data range.

(b) Name one useful additional kind of evidence. [2 marks]

Collect data relevant to the proposed forecast range.

Answer: Measurements over a longer period, including evidence of resource limitations or changing conditions.

9. A normal sensor-error model has mean 0 and SD 0.6 units. [6 marks]

(a) Find $P(-0.9 < X < 0.9)$. [3 marks]

Standardise the limits to -1.5 and 1.5.

Answer: 0.8664.

(b) Find the positive value c for which $P(X < c) = 0.975$. [3 marks]

$c = 0 + 0.6\Phi^{-1}(0.975)$.

Answer: 1.176 units.

10. A normal model for a measurement has mean 80 and SD 5. [5 marks]

(a) Find the probability that a measurement is at least 90. [3 marks]

$z = \frac{90 - 80}{5} = 2$.

Use the upper tail.

Answer: 0.0228.

(b) Out of 500 measurements, find the expected number at least 90. [2 marks]

Multiply the unrounded probability by 500.

Answer: 11.38, about 11.