Residuals

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. For $\hat{y} = 3 + 4x$ and the observation $(2, 13)$, find the residual. [3 marks]

Predicted response is $3 + 8 = 11$.

$\text{Residual} = 13 - 11 = 2.$

Answer: 2.

2. For $\hat{y} = 20 - 0.5x$ and the observation $(6, 15)$, find the residual. [3 marks]

Predicted response is 17.

$\text{Residual} = 15 - 17 = -2.$

Answer: -2.

3. A residual is -4 kg. Interpret its sign and unit. [3 marks]

Residuals retain response units.

Answer: The observation is 4 kg below the prediction, so the model overestimates mass by 4 kg.

4. Residuals are $-0.4, 0.6, -0.2, 0.1, -0.1$. Find their sum. Does a zero sum alone prove a good fit? [3 marks]

Add the signed residuals.

A zero sum can coexist with curvature or other unsuitable patterns.

Answer: $\text{Sum} = 0$; no.

5. Residual spread grows as x increases. What feature needs investigation? [3 marks]

The fit may have non-constant error spread.

A single typical prediction error may not describe the whole range.

Answer: The variability of errors may increase with x.

6. A line was chosen by eye, not fitted by least squares. Must its residuals sum to zero? [3 marks]

A freely chosen line is not constrained by that fitting condition.

Answer: No. The zero-sum property applies to the least-squares intercept fit to the same data.