Least-squares regression

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. Find the line from $\bar{x}=6, \bar{y}=20, S_{xx}=10$ and $S_{xy}=25$. [3 marks]

$b=\frac{25}{10}=2.5.$

$a=20-2.5\times 6=5.$

Answer: $\hat{y}=5+2.5x.$

2. Given $r=-0.6, s_x=5, s_y=10, \bar{x}=8$ and $\bar{y}=30$, find the regression line. [3 marks]

$b=-0.6\times\frac{10}{5}=-1.2.$

$a=30-(-1.2\times 8)=39.6.$

Answer: $\hat{y}=39.6-1.2x.$

3. A candidate line has residuals -2,1,1. Calculate its sum of squared residuals. [3 marks]

Square each residual: 4,1,1.

Add the squares, not their signed values.

Answer: 6.

4. Two models have residuals $A = (-3, 0, 3)$ and $B = (-2, 1, 1)$. Which has the smaller least-squares error? [3 marks]

A has squared sum $9 + 0 + 9 = 18$.

B has $4 + 1 + 1 = 6$, so B is better on this criterion.

Answer: B.

5. A regression line has $a = 7$ and $b = 0.5$. Predict y at $x = 12$. [3 marks]

Substitute $x = 12$ into $\hat{y} = 7 + 0.5x$.

$7 + 6 = 13$.

Answer: 13.

6. A student rounds $b = 0.08473$ to 0.1 before predicting at $x = 900$. Explain the risk. [3 marks]

A small coefficient rounding can become large after multiplication by x.

Keep guard digits until the final prediction.

Answer: The slope contribution changes from 76.257 to 90, an error of 13.743.