# Matrix models - Applications test 6

Name: _____ Date: _____

10 questions | 59 marks | 70 minutes

Linked models, inference and justified recommendations. Network matrices use rows=start and columns=destination; transition matrices use columns=current and rows=next unless stated otherwise.

Use a graphics calculator or appropriate digital tool for labelled tasks.

**1.** A closed two-state matrix has rows $\begin{pmatrix} 0.9 & b \\ 0.1 & 1-b \end{pmatrix}$. A current state $(60, 40)$ is predicted to become $(62, 38)$. [6 marks]

**(a)** Find $b$ and check the resulting matrix. [3 marks]

**(b)** Find steady counts for a total of 300. [3 marks]

**2.** A service-count adjacency matrix has rows $\begin{pmatrix} 0 & k & 1 \\ 0 & 0 & 2 \\ 1 & 0 & 0 \end{pmatrix}$, ordered A,B,C, with $k$ a nonnegative integer. [6 marks]

**(a)** If there are six two-step walks A to C, find $k$. [3 marks]

**(b)** Find the number of three-step A-to-C walks and explain the route. [3 marks]

**3.** Three towns form the directed cycle A→B→C→A. [5 marks]

**(a)** Find which of $A_{AB}, (A^2)_{AB}, (A^3)_{AB}, (A^4)_{AB}$ are nonzero. [3 marks]

**(b)** Explain why a route-planning app must distinguish reachability from a required exact number of transfers. [2 marks]

**4.** A ranking rule counts direct wins and two-step wins. Teams A,B,C form the cycle A beats B, B beats C, C beats A. [5 marks]

**(a)** Show that the rule cannot select a unique winner if all direct wins get weight 1 and all two-step wins the same weight $w$. [3 marks]

**(b)** Give two pieces of additional evidence that could improve a ranking model without merely inventing a tie-break. [2 marks]

**5.** A 100-member population uses transition rows $\begin{pmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{pmatrix}$. The next state is $(60, 40)$. [6 marks]

**(a)** Recover the current state. [4 marks]

**(b)** Verify both destination counts. [2 marks]

**6.** Start at $(100, 0)$. Let $T$ have rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$, and $B$ rows $\begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}$. [7 marks]

**(a)** Apply policies T, then B, then T. Write the product and final state. [4 marks]

**(b)** Compare the final state for policies B, then T, then B. [3 marks]

**7.** A three-state matrix has rows $\begin{pmatrix} 0.5 & a & 0 \\ 0.5 & 1-2a & 0.5 \\ 0 & a & 0.5 \end{pmatrix}$. [5 marks]

**(a)** Find the allowable range of $a$ for a transition matrix. [2 marks]

**(b)** Find $a$ if $(25, 50, 25)$ is steady. [3 marks]

**8.** A two-state policy keeps 85% of current A in A. The proportion of B moving to A is $b$. [7 marks]

**(a)** Choose $b$ so the steady A proportion is 0.7. [4 marks]

**(b)** If $b$ rises to 0.4, find the new steady A proportion and interpret. [3 marks]

**9.** A model uses rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$, initially $(200, 80)$. The observed next state is $(180, 100)$. [5 marks]

**(a)** Find the one-period prediction and maximum absolute error. [3 marks]

**(b)** The model's two-period prediction is exactly the later observation $(176, 104)$. Does this validate the model permanently? [2 marks]

**10.** Options F and S use transition rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$ and $\begin{pmatrix} 0.98 & 0.03 \\ 0.02 & 0.97 \end{pmatrix}$, respectively. Start at $(100, 0)$. Per-period service cost is AUD 5 for A and AUD 1 for B. [7 marks]

**(a)** Compare expected costs using the distribution after three periods. [4 marks]

**(b)** Compare steady-state costs and explain why the preferred short-term option need not follow from equilibrium alone. [3 marks]