Matrix models - Practice test 5

Name: _____ Date: _____

10 questions | 52 marks | 70 minutes

Sales, service networks and operational decisions. Network matrices use rows=start and columns=destination; transition matrices use columns=current and rows=next unless stated otherwise.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A retailer's daily sales rows are $\begin{pmatrix} 3 & 2 \\ 1 & 4 \\ 5 & 0 \end{pmatrix}$ for products P,Q. The cost/price matrix has rows $\begin{pmatrix} 4 & 6 \\ 7 & 10 \end{pmatrix}$, with columns wholesale and retail dollars per item. [5 marks]

(a) Calculate the product and interpret its order. [3 marks]

(b) Find each day's gross margin before other costs. [2 marks]

2. A service-count adjacency matrix has rows $\begin{pmatrix} 0 & 2 & 1 \\ 0 & 0 & 3 \\ 1 & 0 & 0 \end{pmatrix}$, ordered A,B,C. [5 marks]

(a) Count A-to-C walks of exactly two services. [2 marks]

(b) Count A-to-C walks using either one or two services. [2 marks]

(c) Explain why the two-step count need not describe six different town sequences. [1 mark]

3. Rows A,B,C of an adjacency matrix are $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. [5 marks]

(a) Rewrite it using vertex order B,C,A for both rows and columns. [3 marks]

(b) Should the actual number of walks between named towns change? Explain. [2 marks]

4. A,B,C form an undirected path A-B-C. [5 marks]

(a) How many four-step walks go A to C? [3 marks]

(b) Explain why no odd-step walk goes A to C. [2 marks]

5. A beats C and D; B beats A; C,D have no wins. Rank by row sums of $D + wD^2$ with $w \geq 0$. [6 marks]

(a) Find all four scores in terms of w. [3 marks]

(b) Find the weights giving A first, a tie, or B first. [3 marks]

6. Bike transition rows are $\begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}$, with initial counts $(120, 80)$. [6 marks]

(a) Find counts after three days. [3 marks]

(b) Find steady counts and compare. [3 marks]

7. In an observed month, 70 of 100 A members stay A and 30 move B; 20 of 100 B members move A and 80 stay B. [5 marks]

(a) Estimate the transition matrix. [2 marks]

(b) Use it to predict the following month from the observed month's final state. [3 marks]

8. An independent observation is $(72, 28)$. Forecast P is $(70, 30)$, forecast Q is $(60, 40)$. [4 marks]

(a) Compare sums of absolute state errors. [2 marks]

(b) What conclusion is justified, and what conclusion is not? [2 marks]

9. A 500-item system has transition rows $\begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}$. Per-period costs are AUD 2 for A and AUD 5 for B. [5 marks]

(a) Find steady counts. [3 marks]

(b) Find the corresponding expected cost per period. [2 marks]

10. Use the bike model in Question 6. An operational target is an expected A count no greater than 82. [6 marks]

(a) Find the first whole day meeting the target. [4 marks]

(b) Why is rounding day 4’s count to a whole bike before comparing unsafe? [2 marks]