Matrix models - Practice test 5

10 questions | 52 marks | 70 minutes

Sales, service networks and operational decisions. Network matrices use rows=start and columns=destination; transition matrices use columns=current and rows=next unless stated otherwise.

Equivalent correct methods earn the corresponding marks.

1. A retailer's daily sales rows are $\begin{pmatrix} 3 & 2 \\ 1 & 4 \\ 5 & 0 \end{pmatrix}$ for products P,Q. The cost/price matrix has rows $\begin{pmatrix} 4 & 6 \\ 7 & 10 \end{pmatrix}$, with columns wholesale and retail dollars per item. [5 marks]

(a) Calculate the product and interpret its order. [3 marks]

Day 1: wholesale $3(4)+2(7)=26$; retail $3(6)+2(10)=38$.

Day 2: wholesale $1(4)+4(7)=32$; retail $1(6)+4(10)=46$.

Day 3: wholesale $5(4)+0(7)=20$; retail $5(6)+0(10)=30$.

The 3×2 result has one row per day and columns for wholesale cost and retail income.

Answer: $\begin{pmatrix} 26 & 38 \\ 32 & 46 \\ 20 & 30 \end{pmatrix}$; a 3×2 matrix of wholesale cost and retail income for each day.

(b) Find each day's gross margin before other costs. [2 marks]

Subtract wholesale cost from retail income for each row.

Answer: $(12, 14, 10)$ dollars.

2. A service-count adjacency matrix has rows $\begin{pmatrix} 0 & 2 & 1 \\ 0 & 0 & 3 \\ 1 & 0 & 0 \end{pmatrix}$, ordered A,B,C. [5 marks]

(a) Count A-to-C walks of exactly two services. [2 marks]

Two choices A→B combine with three B→C choices.

Answer: 6.

(b) Count A-to-C walks using either one or two services. [2 marks]

Add the direct count 1 to the two-step count 6.

Answer: 7.

(c) Explain why the two-step count need not describe six different town sequences. [1 mark]

Different services can share the same sequence of vertices.

Answer: All six service choices use the same intermediate town B.

3. Rows A,B,C of an adjacency matrix are $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. [5 marks]

(a) Rewrite it using vertex order B,C,A for both rows and columns. [3 marks]

Reorder both axes consistently.

Answer: Rows $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.

(b) Should the actual number of walks between named towns change? Explain. [2 marks]

Read the new positions for the same named endpoints.

Answer: No; consistent relabelling changes positions, not connections.

4. A,B,C form an undirected path A-B-C. [5 marks]

(a) How many four-step walks go A to C? [3 marks]

The (A, C) entry of P^4 is 2.

They are A→B→A→B→C and A→B→C→B→C.

Answer: 2.

(b) Explain why no odd-step walk goes A to C. [2 marks]

An odd number of switches ends at B.

Answer: Every step switches between the end-vertex group {A,C} and the middle group {B}.

5. A beats C and D; B beats A; C,D have no wins. Rank by row sums of $D + wD^2$ with $w \geq 0$. [6 marks]

(a) Find all four scores in terms of w. [3 marks]

Only B gains two indirect wins via A.

Answer: A:2; B:$1 + 2w$; C:0; D:0.

(b) Find the weights giving A first, a tie, or B first. [3 marks]

Compare 2 with $1 + 2w$.

Answer: A first if $0 \leq w < 0.5$; tie at 0.5; B first if $w > 0.5$.

6. Bike transition rows are $\begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}$, with initial counts $(120, 80)$. [6 marks]

(a) Find counts after three days. [3 marks]

Successive A counts are 100,90,85; subtract from total 200 for B.

Answer: $(85, 115)$.

(b) Find steady counts and compare. [3 marks]

Steady ratio is 2:3 for total 200.

Answer: $(80, 120)$; the third-day prediction is not yet at equilibrium.

7. In an observed month, 70 of 100 A members stay A and 30 move B; 20 of 100 B members move A and 80 stay B. [5 marks]

(a) Estimate the transition matrix. [2 marks]

Divide each destination count by its own starting-state total.

Answer: Rows $\begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}$.

(b) Use it to predict the following month from the observed month's final state. [3 marks]

Final A is $70 + 20 = 90$; B is $30 + 80 = 110$.

Apply the estimated matrix.

Answer: Final state $(90, 110)$, next prediction $(85, 115)$.

8. An independent observation is $(72, 28)$. Forecast P is $(70, 30)$, forecast Q is $(60, 40)$. [4 marks]

(a) Compare sums of absolute state errors. [2 marks]

P:$2 + 2$; Q:$12 + 12$.

Answer: P:4; Q:24.

(b) What conclusion is justified, and what conclusion is not? [2 marks]

More independent observations and contextual checks are needed.

Answer: P fits this observation better by this measure; it does not prove P always forecasts better.

9. A 500-item system has transition rows $\begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}$. Per-period costs are AUD 2 for A and AUD 5 for B. [5 marks]

(a) Find steady counts. [3 marks]

Steady proportions are $(0.4, 0.6)$.

Answer: $(200, 300)$.

(b) Find the corresponding expected cost per period. [2 marks]

Calculate $2(200) + 5(300) = 1900$.

Answer: AUD 1900.

10. Use the bike model in Question 6. An operational target is an expected A count no greater than 82. [6 marks]

(a) Find the first whole day meeting the target. [4 marks]

The A count is $80 + 40(0.5)^n$.

At day 4 it is 82.5; at day 5 it is 81.25.

Answer: Day 5.

(b) Why is rounding day 4’s count to a whole bike before comparing unsafe? [2 marks]

Rounding is a reporting step, not a change to the model.

Answer: It could change the threshold decision; compare the unrounded expectation with 82.