Matrix models - Practice test 4

Name: _____ Date: _____

10 questions | 51 marks | 70 minutes

Structure, exceptional behaviour and careful interpretation. Network matrices use rows=start and columns=destination; transition matrices use columns=current and rows=next unless stated otherwise.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. Let A have rows $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, and let I be the identity matrix. [5 marks]

(a) Find AI and IA. [2 marks]

(b) Calculate A^2 and explain why it is not $2A$. [3 marks]

2. A row-start adjacency matrix C has rows $\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}$. [5 marks]

(a) Calculate C^3. [3 marks]

(b) Explain why all different vertices are reachable although $(C^3)_{AB}=0$. [2 marks]

3. A two-vertex network has a loop at each vertex and an arrow A→B. Its adjacency rows are $\begin{pmatrix}1&1\\0&1\end{pmatrix}$. [5 marks]

(a) Find the number of two-step A-to-B walks and list them. [3 marks]

(b) How many three-step A-to-B walks are there? [2 marks]

4. A dominance matrix has rows $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$. A score is the row sum of $D + wD^2$. [5 marks]

(a) Express the three scores in terms of w. [3 marks]

(b) For nonnegative w, can B overtake A under this rule? Explain. [2 marks]

5. A transition matrix is the 2×2 identity. [4 marks]

(a) Find all steady states totalling 100. [2 marks]

(b) Does the model predict a unique long-run split independent of the start? [2 marks]

6. A transition matrix has rows $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, and initial state $(80, 20)$. [5 marks]

(a) Find x_{10} and x_{11}. [2 marks]

(b) Find a steady state with the same total. Does this initial state approach it? [3 marks]

7. Let T have rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$, let S swap A and B, and start at $(80, 20)$. [6 marks]

(a) Apply T first, then S. [3 marks]

(b) Apply S first, then T, and compare. [3 marks]

8. A closed 100-person group uses rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$. The next state is $(68, 32)$. [6 marks]

(a) Recover the initial state. [4 marks]

(b) Verify it by multiplication. [2 marks]

9. An open model first applies rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$, then adds 5 new members to A. Start at $(60, 40)$. [5 marks]

(a) Find the next state and total. [3 marks]

(b) Explain why writing only $x_1 = Tx_0$ would be inadequate. [2 marks]

10. A report claims: “If every column sums to 1, the model is valid and every starting state approaches one accurate prediction.” [5 marks]

(a) Give two different mathematical objections. [4 marks]

(b) Give one empirical objection. [1 mark]