Matrix models - Practice test 3

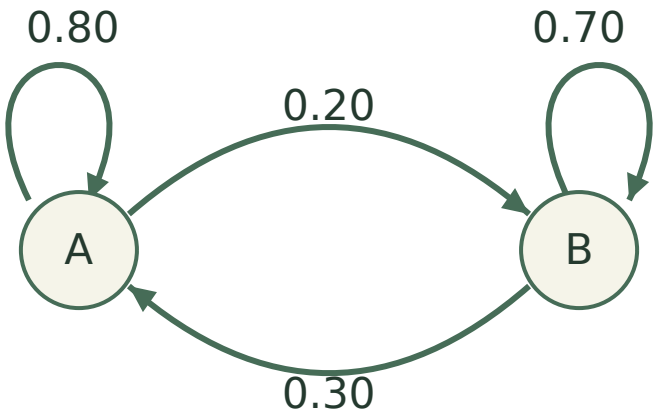
Name: _____ Date: _____

10 questions | 56 marks | 70 minutes

Transition estimation, three-state balance and forecast comparison. Network matrices use rows=start and columns=destination; transition matrices use columns=current and rows=next unless stated otherwise.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. Use the two-state transition diagram. [5 marks]



(a) Write its transition matrix in A,B order with column states. [3 marks]

(b) Explain why adjacency and transition conventions must not be silently interchanged. [2 marks]

2. A survey reports proportions $(0.25, 0.75)$ for a group of 800. Rows of T are $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$. [4 marks]

(a) Write the current count vector. [1 mark]

(b) Predict the next proportions and counts. [3 marks]

3. A month’s movements are supplied below. [6 marks]

Observed transitions: rows=destination, columns=start.

Destination / start	A	B
A	80	15
B	20	35

(a) Estimate a transition matrix using starting-state totals. [3 marks]

(b) Predict next month from the final state of this month. [3 marks]

4. A three-state model has transition rows $\begin{pmatrix} 0.5 & 0.25 & 0 \\ 0.5 & 0.5 & 0.5 \\ 0 & 0.25 & 0.5 \end{pmatrix}$. [6 marks]

(a) Verify it is column-stochastic. [2 marks]

(b) Predict two periods from $(200, 0, 0)$. [4 marks]

5. For rows T: $\begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}$, [6 marks]

(a) Calculate T^2. [3 marks]

(b) Interpret its entry in row A,column B, then predict two periods from $(0, 200)$. [3 marks]

6. Use the three-state matrix in Question 4, with a total of 320. [6 marks]

(a) Find steady counts. [4 marks]

(b) Verify your state by multiplication. [2 marks]

7. A maintenance firm has current state $(160, 40)$. Rows of T are $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$. Next-period cost per item is AUD 4 in A and AUD 7 in B. [6 marks]

(a) Predict the next state. [3 marks]

(b) Find expected next-period cost and compare with applying the same costs to today’s state. [3 marks]

8. A 500-member model uses rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$, starting at $(500, 0)$. [6 marks]

(a) Calculate the state after three periods. [3 marks]

(b) Find steady counts and explain why they differ from the third-period forecast. [3 marks]

9. Compare fast matrix F: $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$, and slow matrix S: $\begin{pmatrix} 0.98 & 0.03 \\ 0.02 & 0.97 \end{pmatrix}$, starting at $(100, 0)$. [7 marks]

(a) Find the steady proportions of both. [3 marks]

(b) Use technology to find the A amount after five periods and explain the difference. [4 marks]

10. Two held-out observations are supplied below. [4 marks]

Observed and predicted counts.

Month	Prediction A	Prediction B	Observed A	Observed B
1	70	30	74	26
2	65	35	66	34

(a) Find the maximum absolute state error in each month. [2 marks]

(b) Explain why fitting exactly to month 1 might worsen model evaluation. [2 marks]