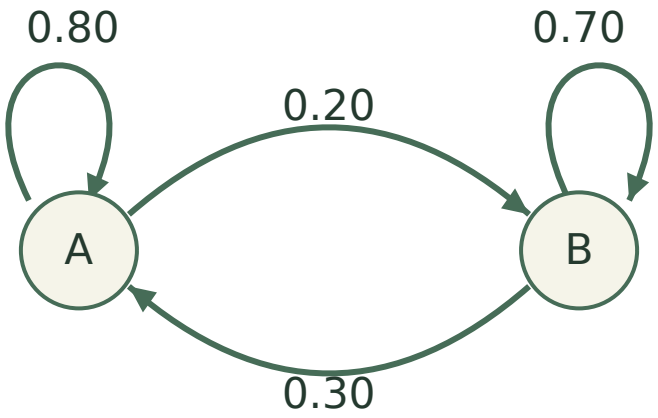
Matrix models - Practice test 3

10 questions | 56 marks | 70 minutes

Transition estimation, three-state balance and forecast comparison. Network matrices use rows=start and columns=destination; transition matrices use columns=current and rows=next unless stated otherwise.

Equivalent correct methods earn the corresponding marks.

1. Use the two-state transition diagram. [5 marks]



(a) Write its transition matrix in A,B order with column states. [3 marks]

The A-start column is $(0.8, 0.2)$; the B-start column is $(0.3, 0.7)$.

Answer: Rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$.

(b) Explain why adjacency and transition conventions must not be silently interchanged. [2 marks]

In this chapter adjacency rows are starts but transition columns are starts.

Answer: The direction attached to an entry changes; label current/next states explicitly.

2. A survey reports proportions $(0.25, 0.75)$ for a group of 800. Rows of T are $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$. [4 marks]

(a) Write the current count vector. [1 mark]

Multiply proportions by 800.

Answer: $(200, 600)$.

(b) Predict the next proportions and counts. [3 marks]

A count is $0.8(200) + 0.3(600) = 340$.

B count is 460; divide by 800.

Answer: Proportions $(0.425, 0.575)$; counts $(340, 460)$.

3. A month's movements are supplied below. [6 marks]

Observed transitions: rows=destination, columns=start.

Destination / start	A	B
A	80	15
B	20	35

(a) Estimate a transition matrix using starting-state totals. [3 marks]

For starts in A divide by 100; for starts in B divide by 50.

Answer: Rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$.

(b) Predict next month from the final state of this month. [3 marks]

Final A is $80 + 15 = 95$ and B is $20 + 35 = 55$.

Apply the estimated matrix to $(95, 55)$.

Answer: Final current state is $(95, 55)$, so next prediction is $(92.5, 57.5)$.

4. A three-state model has transition rows $\begin{pmatrix} 0.5 & 0.25 & 0 \\ 0.5 & 0.5 & 0.5 \\ 0 & 0.25 & 0.5 \end{pmatrix}$. [6 marks]

(a) Verify it is column-stochastic. [2 marks]

Column sums are 1,1,1.

Answer: All entries are nonnegative; each column sums to 1.

(b) Predict two periods from $(200, 0, 0)$. [4 marks]

Multiply once, then again using the complete intermediate vector.

Answer: First $(100, 100, 0)$, then $(75, 100, 25)$.

5. For rows T: $\begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}$, [6 marks]

(a) Calculate T^2. [3 marks]

Row 1: $0.7(0.7) + 0.2(0.3) = 0.55$ and $0.7(0.2) + 0.2(0.8) = 0.30$.

Row 2: $0.3(0.7) + 0.8(0.3) = 0.45$ and $0.3(0.2) + 0.8(0.8) = 0.70$.

Thus T^2 has rows $\begin{pmatrix} 0.55 & 0.30 \\ 0.45 & 0.70 \end{pmatrix}$; each column still sums to 1.

Answer: $\begin{pmatrix} 0.55 & 0.3 \\ 0.45 & 0.7 \end{pmatrix}$

(b) Interpret its entry in row A,column B, then predict two periods from $(0, 200)$. [3 marks]

Multiply the second column of T^2 by 200.

Answer: 0.3 is the two-period B-to-A probability; predicted state $(60, 140)$.

6. Use the three-state matrix in Question 4, with a total of 320. [6 marks]

(a) Find steady counts. [4 marks]

Balance gives $a = c = 0.5b$.

Then $2b = 320$.

Answer: $(80, 160, 80)$.

(b) Verify your state by multiplication. [2 marks]

A: $40 + 40 = 80$; B: $40 + 80 + 40 = 160$; C: $40 + 40 = 80$.

Answer: The matrix sends $(80, 160, 80)$ to itself.

7. A maintenance firm has current state $(160, 40)$. Rows of T are $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$. Next-period cost per item is AUD 4 in A and AUD 7 in B. [6 marks]

(a) Predict the next state. [3 marks]

A: $128 + 12 = 140$; B: $32 + 28 = 60$.

Answer: $(140, 60)$.

(b) Find expected next-period cost and compare with applying the same costs to today's state. [3 marks]

Next: $4(140) + 7(60) = 980$.

Current: $4(160) + 7(40) = 920$.

Answer: Next cost AUD 980; today's distribution costs AUD 920, so an increase of AUD 60.

8. A 500-member model uses rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$, starting at $(500, 0)$. [6 marks]

(a) Calculate the state after three periods. [3 marks]

Scale the states $(80, 20), (70, 30), (65, 35)$ by 5.

Answer: $(325, 175)$.

(b) Find steady counts and explain why they differ from the third-period forecast. [3 marks]

Balance gives ratio 3:2; convergence takes repeated periods.

Answer: Steady $(300, 200)$; three periods need not reach equilibrium.

9. Compare fast matrix F: $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$, and slow matrix S: $\begin{pmatrix} 0.98 & 0.03 \\ 0.02 & 0.97 \end{pmatrix}$, starting at $(100, 0)$. [7 marks]

(a) Find the steady proportions of both. [3 marks]

For F, $0.2a = 0.3b$; for S, $0.02a = 0.03b$.

Answer: Both have steady proportions $(0.6, 0.4)$.

(b) Use technology to find the A amount after five periods and explain the difference. [4 marks]

For F, $a_n = 60 + 40(0.5)^n$.

For S, $a_n = 60 + 40(0.95)^n$.

Answer: Fast: 61.25; slow: about 90.9512. The same equilibrium does not mean the same convergence speed.

10. Two held-out observations are supplied below. [4 marks]

Observed and predicted counts.

Month	Prediction A	Prediction B	Observed A	Observed B
1	70	30	74	26
2	65	35	66	34

(a) Find the maximum absolute state error in each month. [2 marks]

Take absolute differences in each state.

Answer: Month 1:4; month 2:1.

(b) Explain why fitting exactly to month 1 might worsen model evaluation. [2 marks]

A single exact fit is not a guarantee about later observations.

Answer: It could overfit one observation; assess repeated independent evidence and uncertainty.