Matrix models - Practice test 2

Name: _____ Date: _____

10 questions | 45 marks | 70 minutes

Route networks, changing rules and evidence for a model. Network matrices use rows=start and columns=destination; transition matrices use columns=current and rows=next unless stated otherwise.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A sales matrix M has order 3×2 and a column price vector p has order 2×1. [4 marks]

(a) Which of Mp and pM is defined? Give its order. [2 marks]

(b) A student instead multiplies each sale by one price chosen at random. Explain the missing idea. [2 marks]

2. For two locations A,B, rail matrix R has rows $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$, and bus matrix S has rows $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Entries count direct services. [5 marks]

(a) Calculate RS. [3 marks]

(b) How many rail-then-bus walks go A to B? Compare bus-then-rail. [2 marks]

3. A weighted adjacency matrix W has rows $\begin{pmatrix} 0 & 2 & 1 \\ 0 & 0 & 3 \\ 1 & 0 & 0 \end{pmatrix}$, ordered A,B,C. Entries count different direct services. [4 marks]

(a) Find the number of two-service walks from A to C. [2 marks]

(b) Why is this not the number of distinct intermediate towns? [2 marks]

4. A,B,C form an undirected path A-B-C. [5 marks]

(a) Calculate its squared adjacency matrix. [3 marks]

(b) Count A-to-C walks using one, two or three edges. [2 marks]

5. A four-vertex network has arrows A→B, B→C, C→A, C→D. D has no outgoing arrows. [3 marks]

(a) List the vertices reachable from B, excluding B itself. [2 marks]

(b) Can D return to B? State what feature decides this. [1 mark]

6. Three teams form a dominance cycle A beats B, B beats C, C beats A. Score by row sums of $D + 2D^2$. [4 marks]

(a) Calculate each score. [2 marks]

(b) Suggest a fair way to report the result rather than invent a unique winner. [2 marks]

7. A hire firm uses transition rows $\begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}$ and starts with $(80, 40)$ bikes. [5 marks]

(a) Calculate tomorrow's expected state. [3 marks]

(b) Interpret the matrix entry 0.3 and check the total. [2 marks]

8. A population starts at $(100, 0)$. Period 1 uses T with rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$; period 2 uses U with rows $\begin{pmatrix} 0.6 & 0.1 \\ 0.4 & 0.9 \end{pmatrix}$. [6 marks]

(a) Write and calculate the correctly ordered forecast. [3 marks]

(b) Calculate the reversed-order forecast and interpret the difference. [3 marks]

9. A repair model has states Working,Faulty and transition rows $\begin{pmatrix} 1 & 0.2 \\ 0 & 0.8 \end{pmatrix}$. Start with $(0, 50)$. [5 marks]

(a) Find the state after 3 periods. [3 marks]

(b) Find the steady proportions and interpret the model's one-way assumption. [2 marks]

10. A closed model predicts $(60, 40)$, but later observations are $(58, 47)$. [4 marks]

(a) Compare the totals and identify a missing process. [2 marks]

(b) Would fitting a new matrix to this same observation independently validate it? Explain. [2 marks]