Matrix models - Practice test 1

Name: _____ Date: _____

10 questions | 45 marks | 70 minutes

Core techniques, network interpretation and two-state forecasts. Network matrices use rows=start and columns=destination; transition matrices use columns=current and rows=next unless stated otherwise.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. Matrices have rows A: $\begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$, and B: $\begin{pmatrix} 1 & 5 \\ -2 & 0 \end{pmatrix}$. [5 marks]

(a) Find $A + B$. [2 marks]

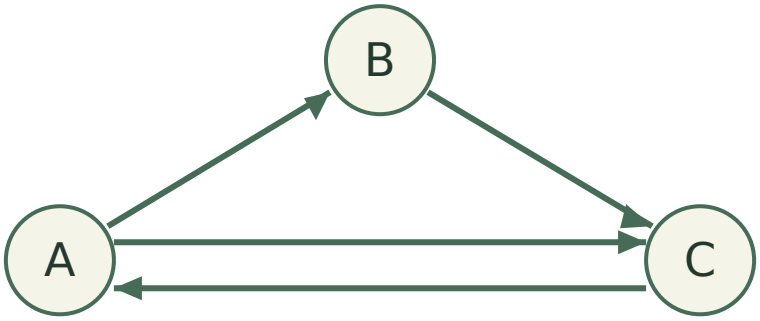
(b) Find $2A - B$. [3 marks]

2. A shop sells two products at AUD 4 and AUD 7. Quantities sold on three days are $(5, 2), (3, 4), (6, 1)$. [5 marks]

(a) State the orders of the sales matrix and price vector. [2 marks]

(b) Use a matrix product to calculate the three daily incomes. [3 marks]

3. The diagram shows direct links, ordered A,B,C. [4 marks]



Each arrow is one permitted move.

(a) Write the adjacency matrix. [3 marks]

(b) State the number of links entering C. [1 mark]

4. Use adjacency rows $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, ordered A,B,C. [5 marks]

(a) Calculate A^2. [3 marks]

(b) Interpret entry row A, column C of A^2 and identify the walk. [2 marks]

5. An unweighted directed network is the chain P→Q→R→S. [4 marks]

(a) Is S reachable from P? Which matrix power first records it? [2 marks]

(b) Can S reach P? Explain. [2 marks]

6. A beats B and C; B beats C; C has no wins. Score by row sums of $D + 0.5D^2$. [5 marks]

(a) Calculate the direct-win scores. [2 marks]

(b) Calculate the weighted scores and rank the teams. [3 marks]

7. A two-state transition matrix has rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$, in A,B order. [4 marks]

(a) Interpret the entry 0.3. [1 mark]

(b) Predict the next state from $(120, 80)$. [3 marks]

8. A proposed closed transition matrix has rows $\begin{pmatrix} 0.6 & p \\ q & 0.75 \end{pmatrix}$. [4 marks]

(a) Find p, q. [2 marks]

(b) Is column summation alone sufficient to check a transition matrix? Explain. [2 marks]

9. For T with rows $\begin{pmatrix}0.8 & 0.3\\0.2 & 0.7\end{pmatrix}$, let $x_0 = (100, 0)$. [4 marks]

(a) Find x_3. [3 marks]

(b) Explain why T^3 alone is not the population forecast. [1 mark]

10. A bike model uses rows $\begin{pmatrix}0.7 & 0.2\\0.3 & 0.8\end{pmatrix}$. There are 250 bikes. [5 marks]

(a) Find steady counts. [3 marks]

(b) Give two assumptions needed for using this as a long-run forecast. [2 marks]